Selling Consumer Data for Profit: 
Optimal Market-Segmentation Design and its Consequences

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Abstract

A data broker sells market segmentations created by consumer data to a producer with private production cost who sells a product to a unit mass of consumers with heterogeneous values. In this setting, I completely characterize the revenue-maximizing mechanisms for the data broker. In particular, every optimal mechanism induces quasi-perfect price discrimination. That is, the data broker sells the producer a market segmentation described by a cost-dependent cutoff, such that all the consumers with values above the cutoff end up buying and paying their values while the rest of consumers do not buy. The characterization of optimal mechanisms leads to additional economically relevant implications. I show that the induced market outcomes remain unchanged even if the data broker becomes more active in the product market by gaining the ability to contract on prices; or by becoming an exclusive retailer, who purchases both the product and the exclusive right to sell the product from the producer, and then sells to the consumers directly. Moreover, vertical integration between the data broker and the producer increases total surplus while leaving the consumer surplus unchanged, since consumer surplus is zero under any optimal mechanism for the data broker.

Keywords: Price discrimination, market segmentation, mechanism design, virtual cost, quasi-perfect segmentation, quasi-perfect price discrimination, surplus extraction, outcome-equivalence

JEL classification:D42, D82, D61, D83, L12

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1 Introduction

1.1 Motivation

The rapid development of informational technology has enabled one to collect, process and analyze vast volumes of consumer data. By the use of consumer data, the scope of price discrimination has moved far beyond its traditional boundaries such as geography, age, or gender. Extensive usage of consumer data allows one to identify many characteristics of consumers that are relevant to the prediction of their values, and therefore to create numerous sorts of market segmentation—a way to split the market demand into several segments by partitioning the consumers’ characteristics—to facilitate price discrimination. Moreover, because of their specialization in information technology, several “data brokers” trade vast amounts of consumer data with retailers, which effectively means these data brokers can create market segmentations and sell them as a product that facilitates price discrimination. For instance, online platforms such as Facebook collect and sell a significant amount of consumers’ personal information, including personal characteristics, traveling plans, lifestyles, and text messages via its own platform. Alternatively, data companies such as Acxiom and Datalogix gather and sell personal information such as government records, financial activities, online activities and medical records to retailers (Federal Trade Commission, 2014).

This paper studies the design of optimal selling mechanisms of a data broker. In this paper, I consider a model where there is one producer with privately known constant marginal cost, who produces and sells a single product to a unit mass of consumers. The consumers have unit demand and the distribution of their values is described by commonly known market demand. Into this environment, I introduce a data broker, who does not know the producer’s marginal cost of production but can sell any market segmentation to the producer via any selling mechanism. As the data broker is uncertain about the production cost, and only affects the product market indirectly by selling consumer data to the producer, it is not obvious how the data broker should sell market segmentations to the producer, what market segmentations will be created, and how the sale of consumer data affects economic welfare and allocative outcomes.

The main result of this paper is a complete characterization of the revenue-maximizing mechanisms for the data broker. The optimal mechanisms feature quasi-perfect price discrimination, which is an outcome where all the purchasing consumers pay exactly their values.

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1In practice, “selling” consumer data can take a wide variety of forms, which include not only traditional physical transactions but also integrated data-sharing agreements/activities. For instance, in a recent full-scale investigation by The New York Times, Facebook has formed ongoing partnerships with other firms, including Netflix, Spotify, Apple and Microsoft, and granted these companies accesses to different aspects of consumer data “in ways that advanced its own interests.” See full news coverage at https://www.nytimes.com/2018/12/18/technology/facebook-privacy.html
although not every consumer with values above the marginal cost buys the product. Theorem 1 shows that every optimal mechanism must create quasi-perfect segmentations described by a cost-dependent cutoff. Namely, all the consumers with values above the cutoff are separated from each other whereas the consumers with values below the cutoff are pooled with the separated high-value consumers. When pricing optimally under this segmentation, the producer only sells to high-value consumers and induces quasi-perfect price discrimination. Furthermore, the cutoff function under any optimal mechanism is exactly the minimum of the (ironed) virtual marginal cost function and the optimal uniform price as a function of marginal cost. With proper regularity conditions, Theorem 2 shows that there is an optimal mechanism where conditional on being below the cutoff, the distribution of consumer values is the same as the market demand in every market segment, meaning that the low-value consumers are pooled uniformly with the separated high-values.

The characterization of the optimal mechanisms further leads to several economically relevant implications. As the defining feature of quasi-perfect price discrimination, under any optimal mechanism, all the consumers pay their values conditional on buying. This implies that the consumer surplus under any optimal mechanism is zero (Theorem 3). In other words, in terms of consumer surplus, it is as if all the information about the consumers’ values is revealed to the producer. The fact that the data broker only affects the product market indirectly via data provision does not benefit the consumers. Furthermore, Theorem 1 also allows a comparison between data brokership and uniform pricing, where no consumer data can be shared. More specifically, I show that data brokership always increases total surplus (Theorem 4), and can even be Pareto-improving compared with uniform pricing if the data broker has to purchase the data from the consumers (before they learn their values, see Theorem 5).

In addition to the welfare implications, another set of relevant questions pertain to how different market regimes would affect market outcomes. More specifically, how would the market outcomes differ if the data broker, instead of merely supplying consumer data to the producers, plays a more active role in the product market? The characterization given by Theorem 1 allows comparisons across several other natural market regimes in addition to data brokership, including vertical integration, where the data broker and the producer merge and all the private information about production cost is revealed; exclusive retail, where the data broker negotiates with the producer and purchases the product, as well as the exclusive right to sell the product, from the producer; and price-controlling data brokership, where the data broker can prescribe what price to charge in addition to providing consumer data. Using the main characterization, I show that vertical integration between the data broker and the producer increases total surplus while leaving the consumer surplus unchanged (Theorem 6). Furthermore, in terms of market outcomes (i.e., data broker’s revenue, producer’s profit,
consumer surplus and the allocation of the product), I show that data brokership is equivalent to both exclusive retail and price-controlling data brokership (Theorem 7).

The rest of this paper is organized as follows. Continuing in this section, several related literatures are discussed. In Section 2, I provide an illustrative example to demonstrate the design of the data broker’s optimal selling mechanism and its implications. The preliminaries and the model are presented in Section 3. In Section 4, I characterize the optimal mechanisms of the data broker. Consequences of consumer-data brokership are discussed in Section 5. Finally, an extension where the feasible market segmentations are limited can be found in Section 6 and some further discussions are in Section 7.

1.2 Related Literature

This paper is related to various streams of literature. In the literature of price discrimination, several theoretical works center around the welfare effects of price discrimination (see, for instance, Varian (1985), Aguirre et al. (2010) and Cowan (2016)) and provide conditions under which third-degree price discrimination increases or decreases total surplus and output. In addition, Bergemann et al. (2015) show that any surplus division between the consumers and a monopolist can be achieved by some market segmentation. In these papers, market segmentation is treated as an exogenous object, whereas in my paper, market segmentation is determined endogenously by a data broker’s revenue-maximization behavior. By contrast, Ali et al. (2020) study the welfare effect of third-degree price discrimination when the consumers can disclose information about their values voluntarily. Furthermore, Wei and Green (2020) study another channel of price discrimination that does not involve market segmentation (i.e., through providing differential information).

Furthermore, this paper is related to the recent literature of the sale of information by a monopolistic information intermediary. Admati and Pfleiderer (1985) and Admati and Pfleiderer (1990) consider a monopoly who sells information about an asset in a speculative market. Bergemann and Bonatti (2015) explore a pricing problem of a data provider who provides data to facilitate targeted marketing. Bergemann et al. (2018) solve a mechanism design problem in which the designer sells experiments to a decision maker who has private information about his belief. Yang (2019) considers a model where an intermediary can provide information about the product to the consumers and charge the seller for such services. Segura-Rodriguez (2020) studies the revenue maximization of a data broker who sells data to firms that differ in the consumer characteristics they wish to forecast.

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2 See also: Haghpanah and Siegel (2020), who further consider segmentations in environments that feature second-degree price discrimination.

3 Relatedly, Acemoglu et al. (2019), Bergemann et al. (2020) and Ichihashi (2020) examine environments where a data broker buys data from the consumers and then sells the consumer data to downstream firms.
Methodologically, this paper is related to the literature of mechanism design and information design (see, for instance, Mussa and Rosen (1978), Myerson (1981), Kamenica and Gentzkow (2011) and Bergemann and Morris (2016)). More particularly, my paper can be regarded as a mechanism design problem where the information structure is also part of the design object (see, for instance, Bergemann and Pesendorfer (2007), Yamashita (2017) and Dworczak (2020)).

Among the aforementioned papers, Bergemann et al. (2015), Bergemann et al. (2018) and Yang (2019) are the closest to my paper. Specifically, Bergemann et al. (2015) explore the welfare implications of different market segmentations, while I introduce a data broker who designs the market segmentation in order to maximize revenue. Bergemann et al. (2018) study an environment where the agent has private information about his prior belief and characterize the optimal mechanism in a binary-action, binary-state case; or in a binary-type case, while my paper studies a revenue maximization problem where the agent’s private information is part of her intrinsic preference and has a rich action space. Also, this paper allows a large class of priors, including those with infinite support. Finally, Yang (2019) solves for optimal mechanisms of an intermediary that can provide information about the product to the consumers, while in this paper I consider the case where an intermediary sells information about the consumers’ values to the producer.

2 An Illustrative Example

To fix ideas, consider the following simple example. A publisher sells an advanced textbook for graduate study. Her (constant) marginal cost of production $c$ is her private information and takes two possible values, $1/4$ or $3/4$, with equal probability. There is a unit mass of consumers with three possible occupations: faculty, undergraduate students, and graduate students. Each of them constitutes $1/3$ of the entire population. It is common knowledge that the textbook has has value $v = 1$ for an undergraduate student, value $v = 2$ for a graduate student and value $v = 3$ for a faculty member. In addition, suppose that among all the undergraduate students, $1/2$ live in houses and $1/2$ live in apartments, whereas all the graduate students live in apartments and all the faculty members live in houses. This economy can be represented by Figure 1, where Figure 1a plots the partitions of the consumers induced by their occupations and residence types and Figure 1b plots the (inverse) market demand $D_0$.

Suppose that there is a data broker who owns all the data about the consumers (e.g., income, medical records, occupations and residential information) and thus is able to provide any partition on the line in Figure 1a to the publisher. How should the data broker sell these data to the publisher? An intuitive guess is that the data broker should sell the most informative data. That is, he should provide the publisher with occupation data so that
each consumer’s value can be fully revealed. Upon receiving such data, the publisher is able to perfectly price discriminate the consumers. The value-revealing data creates a market segmentation that decomposes the market into three market segments, and each market segment enables the publisher to perfectly identify the value of the consumers in that market segment. As a result, if the price of the value-revealing data is \( \tau \) and if the publisher with cost \( c \in \{1/4, 3/4\} \) buys the data, her net profit would be

\[
\frac{1}{3}(1-c) + \frac{1}{3}(2-c) + \frac{1}{3}(3-c) - \tau.
\]

Alternatively, if the publisher with cost \( c \) does not buy any data, she would then charge an optimal uniform price (either 1, 2 or 3, since these are the only possible consumer values) and earn profit

\[
\max \left\{ (1-c), \frac{2}{3}(2-c), \frac{1}{3}(3-c) \right\}.
\]

Therefore, for any \( \tau \), the publisher with cost \( c \) would buy the value-revealing data if and only if

\[
\frac{1}{3}(1-c) + \frac{1}{3}(2-c) + \frac{1}{3}(3-c) - \tau \geq \max \left\{ (1-c), \frac{2}{3}(2-c), \frac{1}{3}(3-c) \right\},
\]

which simplifies to \( \tau \leq (2-c)/3 \). Thus, since \( c \in \{1/4, 3/4\} \), when \( \tau \leq 5/12 \), the publisher would always buy the value-revealing data regardless of her marginal cost. When \( 5/12 < \tau \leq 7/12 \), the publisher would buy the data only if \( c = 1/4 \). Therefore, charging a price \( \tau = 5/12 \) gives the data broker revenue \( 5/12 \) whereas charging a price \( \tau = 7/12 \) gives the data broker revenue \( 7/12 \times 1/2 = 7/24 < 5/12 \). Hence the optimal price for the value-revealing data is \( 5/12 \) and it gives the data broker revenue \( 5/12 \).

However, the data broker can in fact improve his revenue by creating a menu consisting
Figure 2: Market segmentation induced by residential data

of not just the value-revealing data. To see this, consider the following menu of data

\[ M^* = \left\{ \left( \text{residential data, } \tau = \frac{1}{3} \right), \left( \text{value-revealing data, } \tau = \frac{7}{12} \right) \right\}. \]

Notice that the residential data creates a market segmentation with two segments described by two demand functions, \( D_H \) and \( D_A \). Segment \( D_H \) contains all of the consumers with \( v = 3 \) and 1/2 of the consumers with \( v = 1 \) (i.e., those who live in houses), while segment \( D_A \) contains all of the consumers with \( v = 2 \) and 1/2 of the consumers with \( v = 1 \) (i.e., those who live in apartments). Figure 2 plots this market segmentation. From Figure 2, it can be seen that \( D_H + D_A = D_0 \). Moreover, for the publisher with \( c = 1/4 \), the difference in profit between charging price 3 (2) and charging price 1 in segment \( D_H \) (\( D_A \)) is exactly the difference between the area of the darker region and the area of the lighter region depicted in Figure 2. Therefore, since the area of the lighter region is smaller than the area of the darker region, charging a price of 3 (2) is better than charging a price of 1 in segment \( D_H \) (\( D_A \)). Thus, as there are only two possible values in each segment, charging a price of 3 (2) is optimal for the publisher under segment \( D_H \) (\( D_A \)). This is also the case when her cost is \( c = 3/4 \), since the area of the lighter region would decrease and the area of the darker region would remain unchanged when the marginal cost changes from 1/4 to 3/4. As a result, regardless of her marginal cost, the publisher will sell to all the consumers with values \( v = 3 \) and \( v = 2 \) by charging exactly their values upon receiving the residential data.

With this observation, it then follows that when \( c = 1/4 \), the publisher would prefer buying the value-revealing data (at the price of \( \tau = 7/12 \)) whereas when \( c = 3/4 \), the publisher would prefer buying the residential data (at the price of \( \tau = 1/3 \)). Therefore, when menu \( M^* \) is provided, the data broker’s revenue is

\[ (0.5)\frac{1}{3} + (0.5)\frac{7}{12} = \frac{11}{24} > \frac{5}{12}, \]

which is higher than what can be obtained by selling value-revealing data alone. The intuition
behind such an improvement is simple. When selling the value-revealing data alone, the publisher with lower marginal cost retains more rents because the broker would have to incentivize the high-cost publisher to purchase. However, by creating a menu containing both the value-revealing data and the residential data, the data broker can further screen the publisher. To see this, notice that even though the residential data is less informative than the value-revealing data, the only extra benefit of the value-revealing data is for the publisher to be able to price discriminate the consumers with $v = 1$. Thus, when the publisher’s marginal cost is high (i.e., $c = 3/4$), the additional information given by the value-revealing data is less useful to the publisher because the gains from selling to consumers with $v = 1$ are small. In contrast, when the publisher has a low marginal cost (i.e., $c = 1/4$), the value-revealing data is more valuable to the publisher since the gains from selling to consumers with $v = 1$ are larger. Therefore, by providing a menu that contains two different datasets with different prices, the data broker can screen the publisher and extract more revenue from the publisher with lower marginal cost than by selling the value-revealing data alone.

In fact, as it will be shown in Section 4, $\mathcal{M}^*$ is an optimal mechanism of the data broker. The optimal mechanism $\mathcal{M}^*$ has several notable features. First, when $c = 3/4$, the high-value consumers ($v = 2$ and $v = 3$) are separated from each other whereas the low-value consumers ($v = 1$) are pooled together with the high-value consumers. This induces a market outcome where consumers with values $v = 2$ and $v = 3$ are buying the textbook by paying their values, whereas the consumers with $v = 1$ do not buy, even if their value is greater than the publisher’s marginal cost $3/4$. In other words, in order to maximize revenue, the data broker would sometimes discourage (ex-post) efficient trades. Second, all the purchasing consumers are paying exactly their values, which implies that consumer surplus is zero. Finally, even though every purchasing consumer pays their value, the high-cost publisher never learns exactly each individual consumer’s value. These features are not specific to this simple example. In fact, all of them hold in a general class of environments, which will be explored later in this paper.

3 Model

3.1 Notation

The following notation is used throughout the paper. For any Polish space $X$, $\Delta(X)$ denotes the set of probability measures on $X$ where $X$ is endowed with the Borel $\sigma$-algebra and $\Delta(X)$ is endowed with the with weak-* topology. When $X = [x, \bar{x}] \subseteq \mathbb{R}$ is an interval, let $D(X)$ denote the collection of nonincreasing and upper-semicontinuous functions $D : \mathbb{R}_+ \rightarrow [0, 1]$
such that $D(x) = 1$, $D(x^+) = 0$. Since $\mathcal{D}(X)$ and $\Delta(X)$ are bijective,\(^5\) for any $D \in \mathcal{D}(X)$, let $m^D \in \Delta(X)$ be the probability measure associated with $D$ and define the integral

$$\int_A h(x)D(dx) := \int_A h(x)m^D(dx),$$

for any measurable $h : X \to \mathbb{R}$. Then, endow $\mathcal{D}(X)$ with the weak-* topology and the Borel $\sigma$-algebra using this integral (details in Appendix A). Also, write $\text{supp}(D) := \text{supp}(m^D)$.

### 3.2 Primitives

There is a single product, a unit mass of consumers with unit demand, a producer for this product (she), and a data broker (he). Across the consumers, their values $v$ for the product are distributed according to a commonly known probability measure $m^0 \in \Delta(V)$ and thus can be described by a market demand $D_0 \in \mathcal{D} := \mathcal{D}(V)$, where $D_0(p) := m^0([p,\overline{v}])$ is the share of consumers whose values are above $p$ and $V = [\underline{v},\overline{v}] \subset \mathbb{R}_+$ is a compact interval. Each consumer knows their own value. For the rest of the paper, $D_0$ is said to be regular if the function $p \mapsto (p-c)D_0(p)$ is single-peaked on $\text{supp}(D_0)$ for all $c \geq 0$.\(^6\)

The producer has a constant marginal cost of production $c \in C = [\underline{c},\overline{c}] \subset \mathbb{R}_+$ for some $0 \leq \underline{c} < \overline{c} < \infty$. The marginal cost $c$ is private information to the producer and follows a distribution $G$, where $G$ has a density $g > 0$ and induces a virtual (marginal) cost function $\phi_G$, defined as $\phi_G(c) := c + G(c)/g(c)$ for all $c \in C$. Henceforth, $G$ is said to be regular if $\phi_G$ is increasing.

The data broker owns all the consumer data and can create any market segmentation, which is a probability measure $s \in \Delta(\mathcal{D})$ that satisfies the following condition

$$\int_D D(p)s(dD) = D_0(p), \forall p \in V. \quad (1)$$

That is, a segmentation is a way to split the market demand $D_0$ into different market segments.\(^7\) Let $\mathcal{S}$ denote the set of segmentations.

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\(^4\)As a convention, for any function $f$ defined on $\mathbb{R}_+$, $f(x^+)$ denotes the right limit of $f$ at $x$.

\(^5\)This is because for any $D \in \mathcal{D}(X)$, $1 - D$ is nondecreasing and right-continuous.

\(^6\)When $D_0$ is strictly decreasing on $V$, this is equivalent to saying that the marginal revenue function induced by $D_0$ is decreasing.

\(^7\)As illustrated in the motivating example, different consumer data induce different partitions of consumers’ characteristics and thus different ways to split $D_0$ into a collection of demand functions that sum up to $D_0$. Thus, given a market segmentation $s$, each market segment $D \in \text{supp}(s)$ can be interpreted as a group of consumers who have some common characteristics (e.g., house residents). Notice that by allowing the data broker to provide any market segmentation, it is implicitly assumed that the data broker always has enough data to identify each consumer’s value and is able to segment the consumers according to their values arbitrarily. In Section 6, I consider an extension where the data broker has imperfect information about the consumers’ values.
3.3 Timing of the Events

First, the data broker proposes a mechanism, which contains a set of available messages that the producer can send, as well as mappings that specify the market segmentation and the amount of transfers as functions of the messages. Then, the producer decides whether to participate in the mechanism. If the producer does not, she only operates under $D_0$ without any further segmentations and pays nothing. If the producer participates in the mechanism, she sends a message from the message space, pays the associated transfer, and then operates under the associated market segmentation.

Given any segmentation $s \in S$, the producer engages in price discrimination by choosing a price $p \geq 0$ in each segment $D \in \text{supp}(s)$.

To maximize profit, for any segment $D \in \text{supp}(s)$, the producer with marginal cost $c$ solves

$$\max_{p \in \mathbb{R}^+} (p - c)D(p).$$

For any $c \in C$ and any $D \in \mathcal{D}$, let $P_D(c)$ denote the set of optimal prices for the producer with marginal cost $c$ under market segment $D$. As a convention, regard $P$ as a correspondence on $\mathcal{D} \times C$ and if $p$ is a selection for $P$, write $p \in P$.

Furthermore, for any $c \in C$ and any $D \in \mathcal{D}$, let

$$\pi_D(c) := \max_{p \in \mathbb{R}^+} (p - c)D(p)$$

denote the maximized profit of the producer. Also, let

$$\overline{p}_D(c) := \max_{p \in P_D(c)}$$

be the largest optimal price for the producer with marginal cost $c$ under market segment $D$.

For conciseness, let $\overline{p}_0 := \overline{p}_{D_0}$.

Throughout Section 4 and Section 5, I impose the following technical assumption on the market demand $D_0$ and the distribution $G$.

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8It is without loss of generality to restrict attention to posted price mechanisms even though the producer has private information about $c$ when designing selling mechanisms. This is because the environment features independent private values and quasi-linear payoffs, and both the producer’s and the consumers’ payoffs are monotone in their types. By Proposition 8 of Mylovanov and Tröger (2014), it is as if $c$ is commonly known when the producer designs selling mechanisms. Therefore, since the consumers have unit demand, according to Myerson (1981) and Riley and Zeckhauser (1983), it is without loss to restrict attention to posted price mechanisms.

9For notational conveniences, I restrict the feasible prices for each producer to a large enough compact interval $V \subset \mathbb{R}^+$ such that $V \subset \overline{V}$. With this restriction, $P_D(c)$ would be a subset of a compact interval for all $D \in \mathcal{D}$ and for all $c \in C$. Since $V$ itself is bounded, this restriction is simply a notational convention and does not affect the model at all.

10$\overline{p}$ is well-defined under the notational convention stated in footnote 9, as $P_D$ is a closed (implied by upper-semicontinuity of $D$) subset of a compact set $\overline{V}$. 
Assumption 1. The function $c \mapsto \max\{g(c)(\phi_{G}(c) - \bar{p}_0(c)), 0\}$ is nondecreasing.

Assumption 1 permits a wide class of $(D_0, G)$ and includes many common examples.\textsuperscript{11} Also, it does not require regularities of either $D_0$ or $G$ (nor is it implied by regularities of $D_0$ and $G$). In Section 6, I will further discuss this assumption, including how the results rely on it, its relaxations, as well as several economically interpretable sufficient conditions.

3.4 Mechanism

When proposing mechanisms, by the revelation principle (Myerson, 1979), it is without loss to restrict the data broker’s choice of mechanisms to incentive compatible and individually rational direct mechanisms that ask the producer to report her marginal cost and then provide the segmentation and determine the transfer accordingly.\textsuperscript{12}

Formally, a mechanism is a pair $(\sigma, \tau)$, where $\sigma : C \rightarrow S$, $\tau : C \rightarrow \mathbb{R}$ are measurable functions. Given a mechanism $(\sigma, \tau)$, for each $c \in C$, $\sigma(c) \in S$ stands for the market segmentation provided to the producer, $\tau(c) \in \mathbb{R}$ stands for the amount the producer pays to the data broker. Moreover, any measurable $\sigma : C \rightarrow S$ is called a segmentation scheme (or sometimes, a scheme).

A mechanism $(\sigma, \tau)$ is incentive compatible if for all $c, c' \in C$,

$$\int_D \pi_D(c)\sigma(dD|c) - \tau(c) \geq \int_D \pi_D(c)\sigma(dD|c') - \tau(c').$$

(IC)

Also, since the producer can always sell to the consumers by charging a uniform price, a mechanism $(\sigma, \tau)$ is individually rational if for all $c \in C$,

$$\int_D \pi_D(c)\sigma(dD|c) - \tau(c) \geq \pi_{D_0}(c).$$

(IR)

Henceforth, a mechanism $(\sigma, \tau)$ is said to be incentive feasible if it is incentive compatible and individually rational, and a segmentation scheme $\sigma$ is said to be implementable if there exists a measurable $\tau : C \rightarrow \mathbb{R}$ such that $(\sigma, \tau)$ is incentive feasible. The goal of the data broker is to maximize expected revenue $\mathbb{E}_G[\tau(c)]$ by choosing an incentive feasible mechanism.

The data broker’s revenue maximization problem exhibits several noticeable features. First, the object being allocated is infinite dimensional. After all, the data broker sells market segmentations to the producer as opposed to a one-dimensional quality or quantity variable in classical mechanism design problems (e.g., Mussa and Rosen (1978), Myerson (1981) and Maskin and Riley (1984)). In particular, it is not clear whether there exists a

\textsuperscript{11}For instance, if $D_0$ is linear demand and $G$ is uniform; or if both $D_0$ and $G$ are exponential on some intervals; or if $D_0$ and $G$ are such that $D_0(v) = (1 - v)^\beta$, $G(c) = c^\alpha$, for all $v \in [0, 1]$, $c \in [0, 1]$, for any $\alpha, \beta > 0$; or if $D_0$ and $G$ take one of the aforementioned forms.

\textsuperscript{12}Henceforth, unless otherwise noted, a mechanism stands for a direct mechanism.
partial order on the space of market segmentations that would lead to the single-crossing property commonly assumed in low-dimensional screening problems. Second, the producer’s outside option is type-dependent. This is because the producer has access to the consumers, and is only buying the additional information about the consumers’ values.

As another remark, the model introduced above is equivalent to a model where there is one producer with private cost \( c \) and one consumer with private value \( v \), where \( c \) and \( v \) are independently drawn from \( G \) and \( m^0 \), respectively. With this interpretation, a segmentation \( s \in S \) is then equivalent to a Blackwell experiment that provides the producer with information regarding the consumer’s private value. Throughout the paper, the analyses and results are stated in terms of the version with a continuum of consumers, yet every statement and interpretation has an equivalent counterpart in the version with one consumer who has a private value.

4 Optimal Segmentation Design

In what follows, I characterize the data broker’s optimal mechanisms. To this end, I first introduce a crucial class of mechanisms. Then I characterize the optimal mechanisms by this class.

4.1 Quasi-Perfect Segmentations and Quasi-Perfect Price Discrimination

As illustrated in the motivating example, to elicit private information from the producer, the data broker may sometimes wish to discourage sales even when there are gains from trade. In addition, the data broker would wish to extract as much surplus as possible by providing market segmentations under which all the purchasing consumers pay their values. These two features jointly lead to a specific form of market segmentation, which will be referred as quasi-perfect segmentations.

Definition 1. For any \( c \in C \) and any \( \kappa \geq c \), a segmentation \( s \in S \) is a \( \kappa \)-quasi-perfect segmentation for \( c \) if for \( s \)-almost all \( D \in D \), either \( D(c) = 0 \), or the set \( \{ v \in \text{supp}(D) : v \geq \kappa \} \) is a singleton and is a subset of \( P_D(c) \).

A \( \kappa \)-quasi-perfect segmentation for \( c \) is a segmentation that separates all the consumers with \( v \geq \kappa \) while pooling the rest of the consumers together with them so that when the producer’s marginal cost is \( c \), every market segment with positive trading volume\(^{13}\) must contain one and only one consumer-value \( v \geq \kappa \) and this \( v \) is an optimal price for the producer. Notice that a \( \kappa \)-quasi-perfect segmentation for \( c \) induces \( \kappa \)-quasi-perfect price

\(^{13}\)Notice that when the producer’s marginal cost is \( c \), no trade occurs in market segment \( D \) if and only if \( D(c) = 1 \).
**discrimination** when the producer’s marginal cost is \( c \) and when she charges the largest optimal price in (almost) all segments. Namely, a consumer with value \( v \) buys the product if and only if \( v \geq \kappa \) and all purchasing consumers pay exactly their values. For instance, in the example given by Section 2, the residential data creates a 2-quasi-perfect segmentation for both \( c \in \{1/4, 3/4\} \). With Definition 1, I now define the following:

**Definition 2.** Given any function \( \psi : C \to \mathbb{R} \) with \( c \leq \psi(c) \) for all \( c \in C \):

1. A segmentation scheme \( \sigma \) is a \( \psi \)-quasi-perfect scheme if for \( G \)-almost all \( c \in C \), \( \sigma(c) \) is a \( \psi(c) \)-quasi-perfect segmentation for \( c \).

2. A mechanism \( (\sigma, \tau) \) is a \( \psi \)-quasi-perfect mechanism if \( \sigma \) is a \( \psi \)-quasi-perfect scheme and if the producer with marginal cost \( \bar{c} \), when reporting truthfully, has net profit \( \pi_{D_0}(\bar{c}) \).

### 4.2 Characterization of the Optimal Mechanisms

With the definitions above, the main characterization of this paper can be stated. To this end, for any \( c \in C \), define \( \varphi_G(c) := \min\{\varphi_G(c), \bar{p}_0(c)\} \), where \( \varphi_G \) is the ironed virtual cost function.\(^{14}\)

**Theorem 1** (Optimal Mechanism). The set of optimal mechanisms is nonempty and is exactly the set of incentive feasible \( \varphi_G \)-quasi-perfect mechanisms. Furthermore, every optimal mechanism induces \( \varphi_G(c) \)-quasi-perfect price discrimination for \( G \)-almost all \( c \in C \).

From the definition of quasi-perfect segmentations, there are some degrees of freedom regarding the ways to pool the low-value consumers with the high-values. Therefore, Theorem 1 implies that there might be multiple optimal mechanisms—as long as the low-value consumers are pooled with the high-values in a way such that the mechanism is incentive feasible and is \( \varphi_G \)-quasi-perfect. Nevertheless, the outcome induced by any optimal mechanism is unique. That is, under any optimal mechanism, for (almost) all marginal cost \( c \in C \), a consumer with value \( v \) buys the product if and only if \( v \geq \varphi_G(c) \) and all the purchasing consumers pay their values. In other words, the multiplicity only accounts for the off-path incentives. Furthermore, there is always an explicit construction of an optimal mechanism (see details in Appendix B). In fact, when the market demand \( D_0 \) is regular, this construction takes a particularly simple form: The low-value consumers are pooled with the high values in a way that preserves the likelihood ratios among values below the cutoff. More specifically, for any \( c \in C \) and for any \( v \geq \varphi(c) \), define market segment \( D_{\varphi_G(c)}^v \in \mathcal{D} \) as

\[
D_{\varphi_G(c)}^v(p) := \begin{cases} 
D_0(p), & \text{if } p \in [v, \varphi_G(c)] \\
D_0(\varphi_G(c)), & \text{if } p \in (\varphi_G(c), v] \\
0, & \text{if } p \in (v, \bar{v}] 
\end{cases}
\]  

\(^{14}\)Ironing in the sense of Myerson (1981).
for all \( p \in V \). Moreover, for any \( c \in C \) and for any \( p \in [\varphi_G(c), \bar{v}] \), let

\[
\sigma^* \left( \{ D^{\varphi_G(c)}_v : v \geq p \} \mid c \right) := \frac{D_0(p)}{D_0(\varphi_G(c))}.
\]

(3)

In other words, for any \( c \in C \), \( \sigma^*(c) \) only assigns positive measure to market segments \( \{ D^{\varphi_G(c)}_v \}_{v \in [\varphi_G(c), \bar{v}]} \) and its distribution is exactly the distribution of consumers’ values conditional on being above the cutoff \( \varphi_G(c) \) given by the market demand.\(^{15}\) Figure 3 illustrates \( \sigma^* \) by plotting the (inverse) demand of a generic market segment \( D^{\varphi_G(c)}_v \) induced by \( \sigma^*(c) \) (the dashed line represents the market demand \( D_0 \)). This inverse demand has a jump at \( D_0(\varphi_G(c)) \). To the left of \( D_0(\varphi_G(c)) \), all the consumer values are concentrated at \( v \), whereas the distribution of the consumer values to the right of \( D_0(\varphi_G(c)) \) remains the same as that under \( D_0 \).

With this definition, it turns out that when \( D_0 \) is regular, as it will be shown below, there exists a unique transfer scheme \( \tau^* : C \rightarrow \mathbb{R} \) such that \((\sigma^*, \tau^*)\) is an incentive feasible \( \varphi_G \)-quasi-perfect mechanism. Thus, by Theorem 1, \((\sigma^*, \tau^*)\) is optimal. Henceforth, I refer the mechanism \((\sigma^*, \tau^*)\) as the canonical \( \varphi_G \)-quasi-perfect mechanism.

**Theorem 2.** Suppose that \( D_0 \) is regular. Then the canonical \( \varphi_G \)-quasi-perfect mechanism \((\sigma^*, \tau^*)\) is optimal.

In what follows, I will describe the main steps of the proof of Theorem 1 (which also lead to the proof of Theorem 2). Details of the proof can be found in Appendix B. Specifically, I first derive a revenue-equivalence formula and characterize the incentive compatible mechanisms.

\(^{15}\)Notice that \( \sigma^* : C \rightarrow S \) is well-defined and measurable since for all \( c \in C \), \( v \mapsto D^{\varphi_G(c)}_v \) is a measurable function from \( V \) to \( D \) and since \( D_0 \circ \varphi_G : C \rightarrow [0,1] \) is also measurable

\(^{16}\)See Appendix A for the formal definition of inverse demands.
Next, I identify an upper bound $\bar{R}$ for the data broker’s revenue. Then, I construct a feasible mechanism that attains $\bar{R}$, which would in turn imply every incentive feasible $\varphi_G$-quasi-perfect mechanism is optimal. Finally, I argue that any mechanism that gives revenue $\bar{R}$ must be a $\varphi_G$-quasi-perfect mechanism.

To highlight the main insights and avoid unnecessary complications, in this subsection, I impose some further assumptions in addition to Assumption 1. More precisely, throughout the remaining part of Section 4.2, I assume that $D_0$ and $G$ are regular and that

$$\phi_G(c) \leq \bar{p}_0(c), \forall c \in C. \quad (4)$$

Notice that (4) is a sufficient condition for Assumption 1. With these additional conditions, $\varphi_G(c) = \phi_G(c)$ for all $c \in C$ and hence $\varphi_G$ can be replaced by the virtual cost function $\phi_G$. Among these assumptions, regularity of $G$ is purely for conciseness, it can be relaxed by ironing $\phi_G$. Regularity of $D_0$ simplifies the construction of the mechanism that attains $\bar{R}$. Without the regularity of $D_0$, the construction would be more involved and can be found in the Appendix B. Lastly, (4) allows a straightforward construction of the revenue upper bound $\bar{R}$. Without (4), the upper bound $\bar{R}$ may not be attainable and a tighter upper bound would be needed, which will be discussed in Section 5. Also, it is noteworthy that all the lemmas stated in this section do not rely on any of these assumptions, nor on Assumption 1.

The Revenue Equivalence Formula and an Upper Bound for Revenue

Even though the data broker’s problem is more complex comparing to a standard monopolistic screening problem due to the high-dimensionality nature of market segmentations, a revenue-equivalence formula can still be derived by properly invoking the envelope theorem. To see this, notice that for any incentive compatible mechanism $(\sigma, \tau)$, the indirect utility of a producer with marginal cost $c$ is

$$U(c) := \int_D \pi_D(c) \sigma(dD|c) - \tau(c)$$

$$= \max_{c' \in C} \int_D \pi_D(c) \sigma(dD|c') - \tau(c')$$

By the envelope theorem, the derivative of $U$ is simply the partial derivative of the objective function evaluated at the optimum, that is,

$$U'(c) = \int_D \pi_D'(c) \sigma(dD|c).$$

Moreover, since $\pi_D(c)$ is the optimal profit of the producer with marginal cost $c$ under segment $D$, again by the envelope theorem, for all $c \in C$,

$$\pi_D'(c) = -D(\bar{p}_D(c)). \quad (5)$$
Together,

\[ U(c) = U(\overline{c}) + \int_c^{\overline{c}} \left( \int_D D(\overline{p}_D(z))\sigma(dD|z) \right) dz, \forall c \in C. \]

Therefore, under any incentive compatible mechanism \((\sigma, \tau)\), if a producer with marginal cost \(c\) misreports a marginal cost \(c'\) and sets prices optimally, the deviation gain can be written as

\[
U(c) - \left( \int_D \pi_D(c)\sigma(dD|c') - \tau(c') \right)
= \int_D [\pi_D(c) - \pi_D(c')]\sigma(dD|c') - (U(c) - U(c'))
= \int_c^{c'} \left[ \int_D -\pi_D'(z)\sigma(dD|c') - \int_D D(\overline{p}_D(z))\sigma(dD|z) \right] dz
= \int_c^{c'} \left[ \int_D D(\overline{p}_D(z))\sigma(dD|c') - \int_D D(\overline{p}_D(z))\sigma(dD|z) \right] dz
\]

Together, these lead to Lemma 1 below.

**Lemma 1.** A mechanism \((\sigma, \tau)\) is incentive compatible if and only if

1. For all \(c \in C\),

\[
\tau(c) = \int_D \pi_D(c)\sigma(dD|c) - \int_c^{\overline{c}} \left( \int_D D(\overline{p}_D(z))\sigma(dD|z) dz \right) - U(\overline{c}).
\]

2. For all \(c, c' \in C\),

\[
\int_c^{c'} \left( \int_D D(\overline{p}_D(z))(\sigma(dD|z) - \sigma(dD|c')) \right) dz \geq 0.
\]

Furthermore, \(\overline{p}\) can be replaced by any \(p \in P\) for the “only if” part.

The proof of Lemma 1 can be found in Appendix B. It formalizes the heuristic arguments above by using the envelope theorem of Milgrom and Segal (2002). In essence, condition 1 in Lemma 1 is a generalized revenue-equivalence formula stating that the transfer \(\tau\) must be determined by \(\sigma\) up to a constant, whereas condition 2 in Lemma 1 is reminiscent of Lemma 1 of Pavan et al. (2014), and relates to the integral monotonicity condition that arises in various mechanism design problems with multi-dimensional allocation spaces (see, for instance, Rochet (1987), Carbajal and Ely (2013), Pavan et al. (2014)).

From Lemma 1, for any incentive compatible mechanism \((\sigma, \tau)\), the data broker’s expected revenue can be written as

\[
\mathbb{E}_G[\tau(c)] = \int_C \left( \int_D (\overline{p}_D(c) - \phi_G(c)) D(\overline{p}_D(c))\sigma(dD|c) \right) G(dc) - U(\overline{c}), \tag{6}
\]
which can be interpreted as the expected virtual profit net of a constant. That is, maximizing the data broker’s expected revenue by choosing an incentive feasible mechanism \((\sigma, \tau)\) is equivalent to maximizing the expected virtual profit—the profit of the producer if her marginal cost \(c\) is replaced by the virtual marginal cost \(\phi_G(c)\) but she still prices optimally according to marginal cost \(c\)—by choosing an implementable scheme \(\sigma\).

With (6), there is an immediate upper bound for the data broker’s revenue. To see this, first notice that since the producer’s outside option is \(\pi_{D_0}(c)\) when her cost is \(c\), for an incentive compatible mechanism \((\sigma, \tau)\) to be individually rational, it must be that \(U(c) \geq \bar{\pi} := \pi_{D_0}(c)\). Moreover, notice that for any \(c \in C\),

\[
\int_{D} (\overline{p}_D(c) - \phi_G(c)) D(\overline{p}_D(c)) \sigma(dD|c) \leq \int_{D} \max_{p \in \mathbb{R}_+} [(p - \phi_G(c)) D(p)] \sigma(dD|c)
\]

\[
\leq \int_{\{v \geq \phi_G(c)\}} (v - \phi_G(c)) D_0(dv),
\]

where the second inequality holds because the last term is the total gains from trade in the economy when the producer’s marginal cost is \(\phi_G(c)\). Together with (6), it then follows that

\[
\bar{R} := \int_{C} \left( \int_{\{v \geq \phi_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc) - \bar{\pi}
\]

\[
\geq \int_{C} \left( \int_{D} (\overline{p}_D(c) - \phi_G(c)) D(\overline{p}_D(c)) \sigma(dD|c) \right) G(dc) - U(\tau)
\]

\[
= \mathbb{E}_{G}[\tau(c)].
\]

In other words, the upper bound \(\bar{R}\) is constructed by ignoring the individual rationality constrains, the global incentive compatibility constraints (i.e., condition 2 in Lemma 1) and by compelling the producer to charge prices that are optimal when her marginal cost is replaced by the virtual marginal cost.

**Attaining \(\bar{R}\)**

By the definition of quasi-perfect segmentations, for any nondecreasing function \(\psi : C \rightarrow \mathbb{R}_+\) and for any \(\psi\)-quasi-perfect scheme \(\sigma\), given any truthful report \(c \in C\), \(\sigma(c)\) must induce \(\psi(c)\)-quasi-perfect price discrimination when the producer charges the largest optimal price in (almost) every segment. By definition, this means that all the consumers with \(v \geq \psi(c)\) would buy the product by paying exactly their values whereas all the consumers with values \(v < \psi(c)\) would not buy. As a result, all the surplus of consumers with \(v \geq \psi(c)\) would be extracted and the trade volume must be the share of consumers with \(v \geq \psi(c)\).\(^{17}\) Namely, for all \(c \in C\), it must be that

\[
\int_{D} \overline{p}_D(c) D(\overline{p}_D(c)) \sigma(dD|c) = \int_{\{v \geq \psi(c)\}} v D_0(dv)
\]

\(^{17}\)Formal arguments are in the proof of Lemma 5 in Appendix B, which can be found in the Online Appendix.
and
\[ \int_D D(\bar{p}_D(c))\sigma(\text{d}D|c) = D_0(\psi(c)). \tag{8} \]

Therefore, if there is an incentive feasible \( \phi_G \)-quasi-perfect mechanism \((\sigma, \tau)\), then by Lemma 1, the data broker can attain revenue
\[
\mathbb{E}[\tau(c)] = \int_C \left( \int_D (\bar{p}_D(c) - \phi_G(c))D(\bar{p}_D(c))\sigma(\text{d}D|c) \right) G(\text{d}c) - \bar{\pi} \\
= \int_C \left( \int_{\{v \geq \phi_G(c)\}} (v - \phi_G(c))D_0(\text{d}v) \right) G(\text{d}c) - \bar{\pi} \tag{9} \\
= \tilde{R}.
\]

However, not every quasi-perfect scheme is implementable. To ensure incentive compatibility, the integral inequality given by condition 2 in Lemma 1 must be satisfied. While this condition involves a continuum of constraints and is difficult to check, the following lemma provides a simpler sufficient condition.

**Lemma 2.** For any nondecreasing function \( \psi : C \to \mathbb{R}_+ \) with \( \psi(c) \geq c \) for all \( c \in C \), and for any \( \psi \)-quasi-perfect scheme \( \sigma \), there exists a transfer scheme \( \tau : C \to \mathbb{R} \) such that \((\sigma, \tau)\) is incentive compatible if for any \( c \in C \),
\[
\psi(z) \leq \bar{p}_D(z), \tag{10}
\]
for (Lebesgue)-almost all \( z \in [c, c] \) and for all \( D \in \text{supp}(\sigma(c)) \).

In essence, Lemma 2 is a sufficient condition that reduces the integral inequalities in Lemma 1 to pointwise inequalities. Details about the proof can be found in Appendix B. The crucial step is to notice that for a \( \psi \)-quasi-perfect scheme, there are always no downward-deviation incentives (i.e., a producer with cost \( c \) would never have an incentive to misreport \( c' < c \)), as a higher-cost producer would find the gains from reducing the cutoff less beneficial than the increment in transfer. With this observation, as the pointwise condition (10) is sufficient to rule out upward-deviation incentives, Lemma 2 then follows.

After simplifying the incentive constraints, the following lemma then provides a crucial sufficient condition for there to exist an incentive compatible \( \psi \)-quasi-perfect mechanism.

**Lemma 3.** For any nondecreasing function \( \psi : C \to \mathbb{R}_+ \) such that that \( c \leq \psi(c) \leq \bar{p}_0(c) \) for all \( c \in C \), there exists a \( \psi \)-quasi-perfect scheme \( \sigma \) that satisfies (10).

A direct consequence of Lemma 2 and Lemma 3 is that there exists an incentive compatible \( \phi_G \)-quasi-perfect mechanism \((\sigma, \tau)\), provided that \( G \) is regular and (4) holds. Furthermore, for any \( c \in C \), (4) also implies that
\[
\int_c^z D_0(\phi_G(z)) \text{d}z \geq \int_c^z D_0(\bar{p}_0(z)) \text{d}z.
\]
Together, by Lemma 1 and (5), after possibly adding a constant to \( \tau \) so that the indirect utility of the producer with cost \( \bar{c} \) equals to \( \bar{\pi} \), \((\sigma, \tau)\) is an incentive feasible \( \phi_G \)-quasi-perfect mechanism, which in turn implies that \((\sigma, \tau)\) is optimal. Together with (9), it then follows that any incentive feasible \( \phi_G \)-quasi-perfect mechanism is optimal.

The proof of Lemma 3 is by construction. For arbitrary \( D_0 \in \mathcal{D} \), the desired segmentation scheme is constructed by first approximating \( D_0 \) with a sequence of step functions \( \{D_n\} \subseteq \mathcal{D} \) that converges to \( D_0 \), followed by finding a desired \( \psi \)-quasi perfect scheme \( \sigma_n \) of each \( D_n \). Together with a continuity property of quasi-perfect mechanisms and optimal prices, the limit of \( \{\sigma_n\} \) is then a desired \( \psi \)-quasi-perfect scheme. Detailed arguments for this general case can be found in the Online Appendix. Here, I provide a simpler proof for the case where \( D_0 \) is regular.

Proof of Lemma 3 (regular \( D_0 \)). For any \( c \in C \) and for any \( v \in [\psi(c), \bar{v}] \), let \( D_v^{\psi(c)} \in \mathcal{D} \) be defined as (2) with \( \overline{\varphi}_G(c) \) being replaced by \( \psi(c) \). Also, let \( \sigma^*: C \to \Delta(\mathcal{D}) \) be defined as (3) with \( \overline{\varphi} \) being replaced by \( \psi \). By construction, \( \sigma^*(c) \in \mathcal{S} \) for all \( c \in C \). Furthermore, \( \sigma^* \) is a \( \psi \)-quasi-perfect scheme satisfying (10). To see this, consider any \( c \in C \). By regularity of \( D_0 \) (i.e., singled-peakedness of \( p \mapsto (p - c)D_0(p) \)) and by the hypothesis that \( \psi(c) \leq \overline{p}_0(c) \), for any \( c \in C \), it must be that \((p - c)D_0(p) \leq (p^{\psi(c)} - c)D_0(p^{\psi(c)}) \) for all \( p \leq \psi(c) \), where \( p^{\psi(c)} := \max\{v \in \text{supp}(D_0) : v \leq \psi(c)\} \). Therefore, for any \( v \geq \psi(c) \), since \( D_0(p^{\psi(c)}) = D_0(\psi(c)) = D_v^{\psi(c)} \) and since \( D_v^{\psi(c)}(p) = D_0(p) \) for all \( p < \psi(c) \), it must be that

\[
(p - c)D_v^{\psi(c)}(p) = (p - c)D_0(p) \leq (p^{\psi(c)} - c)D_0(p^{\psi(c)}) \leq (v - c)D_0(\psi(c)) = (v - c)D_v^{\psi(c)}(v).
\]

Furthermore, when the producer has marginal cost \( z < c \), for any \( v \geq \psi(c) \), since \( \overline{p}_{D_v^{\psi(c)}}(z) = v \) or \( \overline{p}_{D_v^{\psi(c)}}(z) < \psi(c) \). In the former case, since \( \psi \) is nondecreasing, it then follows that \( \overline{p}_{D_v^{\psi(c)}}(z) = v \geq \psi(c) \geq \psi(z) \), as desired. In the latter case, since \( D_v^{\psi(c)}(p) = D_0(p) \) for all \( p \leq \psi(c) \) and since \( p \mapsto (p - z)D_0(p) \) is singled-peaked (by regularity), \( \overline{p}_{D_v^{\psi(c)}}(z) \) must have been the largest optimal price for the producer under \( D_0 \) as well. That is, \( \overline{p}_{D_v^{\psi(c)}}(z) = \overline{p}_0(z) \). Combining with the hypothesis that \( \psi(z) \leq \overline{p}_0(z) \), this then implies that \( \psi(z) \leq \overline{p}_{D_v^{\psi(c)}}(z) \), as desired. As a result, \( \sigma^* \) is indeed a \( \psi \)-quasi-perfect scheme satisfying (10).

Combining Lemma 1, Lemma 2 and Lemma 3, it then follows that there exists an incentive feasible \( \phi_G \)-quasi-perfect mechanism and hence the data broker can attain revenue \( \tilde{R} \), proving the first part of Theorem 1 (under the regularity assumptions and (4)). In fact, even without the assumptions that \( G \) is regular and that (4) holds, as long as \( D_0 \) is regular, the proof above still implies that the canonical \( \overline{\varphi}_G \)-quasi-perfect scheme \( \sigma^* \) defined in (3) is incentive feasible, which in turn leads to the proof of Theorem 2 in the general case.
Uniqueness
To see why any optimal mechanism of the data broker is a $\phi_G$-quasi-perfect mechanism, suppose that $(\sigma, \tau)$ is optimal. Then,

$$
\bar{R} = \int_C \left( \int_{\{v \geq \phi_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc) - \bar{\pi}
= \int_C \left( \int_D (\bar{p}_D(c) - \phi_G(c)) D(\bar{p}_D(c)) \sigma(dD|c) \right) G(dc) - \bar{\pi},
$$

(11)

which in turn implies that for (almost) all $c \in C$,

$$
\int_{\{v \geq \phi_G(c)\}} (v - \phi_G(c)) D_0(dv) = \int_D (\bar{p}_D(c) - \phi_G(c)) D(\bar{p}_D(c)) \sigma(dD|c),
$$

(12)

since the left-hand side is the efficient surplus in an economy where the producer’s cost is $\phi_G(c)$ and hence must be an upper-bound of the right-hand side, and (11) implies that the right-hand side must attain this upper bound.

It then follows that $\sigma$ must be a $\phi_G$-quasi-perfect mechanism. Indeed, if $\sigma$ is not a $\phi_G$-quasi-perfect scheme, it must be that there is a positive $G$-measure of $c \in C$ and a positive $\sigma(c)$-measure of $D \in \text{supp}(\sigma(c))$ such that either $D(v) > 0$ for some $v > \bar{p}_D(c)$, or $D(\phi_G(c)) \neq D(\bar{p}_D(c))$. That is, either there are some consumers with $v \geq \phi_G(c)$ who do not buy the product or buy the product at a price below $v$, or there are some consumers with $v < \phi_G(c)$ who end up buying the product. This contradicts (12). As a result, $(\sigma, \tau)$ must be a $\phi_G$-quasi-perfect mechanism. Moreover, $(\sigma, \tau)$ must also induce quasi-perfect price discrimination since $\bar{p}$ can be replaced with any $p \in P$ according to Lemma 1.

4.3 Further Remarks and Implementation
Theorem 1 underlines a noteworthy feature of the optimal mechanisms. According to Theorem 1, for any optimal mechanism $(\sigma, \tau)$, the segmentation scheme $\sigma$ does not generate value-revealing segmentations in general. Specifically, for any report $c$ such that $\varphi_G(c) > \varphi$, there are market segments $D \in \text{supp}(\sigma(c))$ containing consumers with distinct values. The reason is that in order to incentivize the producer to set prices in desirable ways and to elicit information from the producer, some market segments must contain consumers with values below the desirable threshold $\varphi_G(c)$. By pooling the high-value consumers with the low-value ones in the same market segment while separating them from other high-value consumers, the data broker is able to incentivize the producer to set prices at the highest value in each market segment and induce $\varphi_G(c)$-quasi-perfect price discrimination for all $c$, which in turn enables the data broker to elicit private information by discouraging trade and extract surplus from the purchasing consumers at the same time. This also means it is not optimal for the data broker to release all the information about consumers’ values.
As an example for an optimal mechanism, consider the case where \( D_0 \) is linear and \( G \) is a uniform distribution with \( V = C = [0, 1] \). It then follows \( \varphi_G(c) = 2c \) for all \( c \in [0, 1/3] \) and \( \varphi_G(c) = (1 + c)/2 \) for all \( c \in (1/3, 1] \). In this case, the canonical \( \varphi_G \)-quasi-perfect mechanism is described by a uniform distribution on the market segments \( \{ D_{\varphi G(v)}(c) \}_{v \in [\varphi_G(c), 1]} \), where each market segment \( D_{\varphi G(v)}(c) \) is defined by (2). As another example, notice that in the motivating example (Section 2), the optimal menu \( M^* \), which consists of the value-revealing data (with a price of \( 7/12 \)) and the residential data (with a price of \( 1/3 \)), implements the canonical quasi-perfect mechanism with a desirable cutoff function. Indeed, the residential data induces a 2-quasi-perfect segmentation for \( c = 3/4 \) as it only separates the high-value consumers (graduate students and professors) and pools the low-value consumers (undergraduate students) with them while preserving their mass. On the other hand, the value-revealing data induces a 1-quasi-perfect segmentation for \( c = 1/4 \). According to the characterization above, since market demand \( D_0 \) is regular and since the virtual costs are \( 1/4 \) and \( 5/4 \) (for costs \( 1/4 \) and \( 3/4 \), respectively),\(^{18}\) the menu \( M^* \) is indeed optimal.

Finally, recall that the upper bound \( \bar{R} \) is derived by (i) ignoring the global incentive constraints (i.e., condition 2 of Lemma 1); (ii) compelling the producer to charge prices that are optimal with respect to the virtual cost \( \phi_G(c) \), as opposed to her true cost \( c \); and (iii) ignoring the individual rationality constraints. As shown above, under (4) and regularity assumptions for both \( D_0 \) and \( G \), all these three constraints end up not binding under the optimal mechanism (\( \sigma^*, \tau^* \)). While it is a general feature that (i) and (ii) never bind even without these simplifying assumptions, the mechanism constructed above might violate the individual rationality constraints (iii) when (4) fails. Therefore, another (tighter) upper bound needs to be considered when extending the arguments above to the case when (4) does not necessarily hold, which will be discussed at the end of Section 5.

5 Consequences of Consumer-Data Brokership

5.1 Surplus Extraction

One of the most pertinent questions about consumer-data brokership is how it affects consumer surplus. Are the data broker’s possession of consumer data and the ability to sell them to a producer detrimental for the consumers? If so, to what extent? Meanwhile, can the consumers benefit from the fact that the data broker does not have access to the consumers and only affects the product market indirectly by selling data to the producer? While currently being a focus of policy debates, the following result, as an implication of Theorem 1, answers

\(^{18}\)Although the characterization is stated for cost distributions that admit densities, as in standard mechanism design problems, there is a straightforward analogous notion of virtual cost function when the cost distribution has atoms.
a certain aspect of this question.

**Theorem 3** (Surplus Extraction). *Consumer surplus is zero under any optimal mechanism.*

Theorem 3 follows directly from the characterization given by Theorem 1. Indeed, according to Theorem 1, any optimal mechanism must induce $\varphi_G(c)$-quasi-perfect price discrimination for (almost) all $c \in C$, which means that every purchasing consumer must be paying their values. Notably, Theorem 3 provides an unambiguous and substantial assertion about the consumer surplus under data brokership. According to Theorem 3, even though the data broker does not sell the product to the consumers directly and only affects the market by creating market segmentations for the producer, it is as if the consumers are perfectly price discriminated and all the surplus is extracted away (even though the optimal mechanisms do not induce perfect price discrimination in general). This means that as long as the data broker possesses consumer data and can sell them to a producer, from the consumers’ perspective, it is the same as buying the product from a monopolist who can implement perfect price discrimination. More practically, this result means it is impossible to expect the consumers to benefit from the gap between the ownership of production technology and ownership of consumer data.

5.2 Comparisons with Uniform Pricing

Although Theorem 3 indicates data brokership is undesirable for the consumers, it does not imply that data brokership is detrimental to the entire economy. After all, by facilitating price discrimination, data brokership may increase total surplus comparing to uniform pricing where no information about the consumers’ values is revealed. Theorem 1, together with Proposition 1, allows such a comparison.

**Proposition 1.** *The data broker’s optimal revenue is no less than the consumer surplus under uniform pricing.*

An immediate consequence of Proposition 1 is that total surplus under data brokership is greater compared with uniform pricing, as summarized below.

**Theorem 4** (Total Surplus Improvement). *Data brokership always increases total surplus compared with uniform pricing.*

Theorem 4 means that even though data brokership is extremely harmful to the consumers, in terms of total surplus it creates, however, it is always better than the environment where no information about the consumers’ values can be disclosed.

Another implication of Proposition 1 pertains to the source of consumer data. So far, it has been assumed that the data broker owns all the consumer data and is able to perfectly
predict each consumer’s value. In contrast, a different ownership structure of consumer data can be considered. In this setting, the data broker does not have any data in the first place and has to purchase them from the consumers.\textsuperscript{19} Proposition 1 immediately implies that, if the data broker has to purchase data by compensating the consumers with monetary transfers \textit{before} they learn their values,\textsuperscript{20} then the optimal mechanism would be to purchase all the data by paying the consumers their ex-ante surplus under uniform pricing and then use any optimal mechanism characterized by Theorem 1 to sell these data to the producer. Furthermore, since the data broker’s revenue is greater than the consumer surplus under uniform pricing according to Proposition 1, and since the producer always has an outside option of uniform pricing, this outcome is in fact Pareto improving compared with uniform pricing in the ex-ante sense, as stated below.\textsuperscript{21}

\textbf{Theorem 5} (Data Ownership). \textit{If the consumers own their data and if the data broker can purchase data from the consumers before they learn their values, then data brokership is Pareto improving compared with uniform pricing in the ex-ante sense.}

5.3 Comparisons across Market Regimes

In addition to its welfare implications, the characterization of Theorem 1 provides further insights about the comparisons across different regimes of the market. Indeed, other than selling consumer data to the producer, there are several other market regimes under which the data broker can profit from the consumer data he owns. Therefore, it would be policy-relevant to compare the outcomes induced by these different market regimes. In what follows, I consider several other market regimes in addition to data brokership, including \textit{vertical integration}, \textit{direct acquisition}, \textit{exclusive retail}, and \textit{price-controlling data brokership}. Then, I

\textsuperscript{19}For simplicity, a “purchase” of data here means that the data broker gains access to \textit{all} the consumer data, in the sense that he can provide any segmentation of $D_0$ to the producer once he makes the purchase. In an earlier version of this paper (Yang, 2020c), I further extend the model and allow the data broker to make a take-it-or-leave-it offer to purchase \textit{any} kind of consumer data and then sell them to the producer. (i.e., offer any segmentation of $D_0$ that is a mean-preserving contraction of the segmentation induced by the purchased data.)

\textsuperscript{20}It is crucial here the data broker purchases \textit{before} the consumers learn their value, since otherwise he would also have to screen the consumers to elicit their private information. Such ex-ante purchase of consumer data is plausibly suitable for online activities. After all, in online settings, consumers often do not consider their values about a particular product when they agree that their personal data such as browsing histories, IP address and cookies, can be collected by the data brokers. Nevertheless, other purchase timing would also be a relevant question, which can be explored in future research.

\textsuperscript{21}Jones and Tonetti (2020) also conclude that granting consumers ownership of their own data is welfare-improving. However, their results are derived in a monopolistic competition setting and the main driving force is the non-rival property of data, whereas Theorem 5 is derived under a monopoly setting and the main rationale is that consumer data facilitate price discrimination, which in turn enhance efficiency.
compare the implications among these different regimes using the characterization provided by Theorem 1.

**Vertical Integration**— The producer’s marginal cost of production becomes common knowledge (for exogenous reasons such as regulation or technological improvements) and the data broker vertically integrates with the producer. That is, the vertically integrated entity is able to produce the product and sell to the consumers via perfect price discrimination.

**Exclusive Retail**— The producer’s marginal cost of production remains private. The data broker negotiates with the producer to purchase the product as well as the exclusive right to sell the product. That is, the data broker can offer a menu, where each item in this menu specifies the quantity \( q \in [0, 1] \) that the producer has to produce and supply to the data broker, as well as the amount of payment the data broker has to pay to the producer \( t \). If the producer chooses an item \((q, t)\) from this menu, she receives profit \( t - cq\) while the data broker pays \( t \) and can sell at most \( q \) units exclusively to the consumers through any market segmentation and at any prices. If the producer rejects this menu, she retains her optimal uniform profit and the data broker receives zero.

**Price-Controlling Data Brokership**— The producer’s marginal cost of production is private information. The data broker, in addition to being able to create market segmentations and sell them to the producer, can further specify what price should be charged in each market segment as a part of the contract. If the producer rejects, she retains her optimal uniform pricing profit and the data broker receives zero. That is, the data broker offers a mechanism \((\sigma, \tau, \gamma)\) such that for all \( c, c' \in C \),

\[
\int_{D \times \mathbb{R}_+} (p - c)D(p)\gamma(dp|D, c)\sigma(dD|c) - \tau(c) \geq \int_{D \times \mathbb{R}_+} (p - c)D(p)\gamma(dp|D, c')\sigma(dD|c') - \tau(c')
\]

and for all \( c \in C \),

\[
\int_{D \times \mathbb{R}_+} (p - c)D(p)\gamma(dp|D, c)\sigma(dD|c) - \tau(c) \geq \pi_{D_0}(c),
\]

where for each \( c \in C \), \( \sigma(c) \in S \) is the market segmentation provided to the producer, \( \tau(c) \in \mathbb{R} \) is the payment from the producer to the data broker, and \( \gamma(c) : D \to \Delta(\mathbb{R}_+) \) is a transition kernel so that \( \gamma(\cdot|D, c) \) specifies the distribution which prices charged in segment \( D \) must be drawn from.

With these definitions, for each market regime, there is an associated profit maximization problem. Henceforth, two market regimes are said to be outcome-equivalent if every solution of the profit maximization problems associated with either market regime induces the same market outcome (i.e., consumer surplus, producer’s profit, data broker’s revenue and the allocation of the product).
An immediate consequence of Theorem 1 is the comparison between data brokership and vertical integration. To see this, recall that any optimal mechanism \((\sigma, \tau)\) of the data broker must induce \(\varphi_G\)-quasi-perfect price discrimination but not perfect price discrimination in general, as \(\varphi_G(c) > c\) for all \(c > \underline{c}\). Thus, whenever there are some consumers with values between \(c\) and \(\varphi_G(c)\) for a positive measure of \(c\), any optimal mechanism would not lead to an efficient allocation because there would be some consumers who end up not buying the product even though their values are greater than the marginal cost. Together with Theorem 3, this means that vertical integration between the data broker and producer increases total surplus while leaving the consumer surplus unchanged when \(D_0\) has full support on \(V\) and there is no common knowledge of gains from trade. After all, consumer surplus is always zero under both regimes, whereas the integrated entity after vertical integration does not create any friction and would perfectly price-discriminate the consumers whose values are above the marginal cost.

**Theorem 6 (Vertical Integration).** Compared with data brokership, vertical integration increases total surplus and leaves the consumer surplus unchanged if \(D_0\) is strictly decreasing and \(\underline{v} < \bar{c}\).

To compare other market regimes, it is noteworthy that since prices are contractable under price-controlling data brokership, for any mechanism \((\sigma, \tau, \gamma)\), the producer’s private marginal cost affects her profit only through the quantity produced and sold to the consumers induced by \((\sigma, \gamma)\). This effectively reduces allocation space under price-controlling data brokership to a one-dimensional quantity space, which is the same as the allocation space under exclusive retail. In fact, as stated in Lemma 4 below, price-controlling data brokership is always equivalent to exclusive retail.

**Lemma 4.** Exclusive retail and price-controlling data brokership are outcome-equivalent.

With Lemma 4, to compare exclusive retail and price-controlling data brokership with data brokership, it suffices to compare only price-controlling data brokership with data brokership. This comparison is particularly convenient since the price-controlling data broker’s revenue maximization problem is a relaxation of the data broker’s. After all, with the extra ability to contract on prices, the constraints in the price-controlling data broker’s problem must be weaker. Nevertheless, as an implication of Theorem 1 and Proposition 2 below, it turns out that the data broker’s optimal revenue is in fact the same as the price-controlling data broker’s optimal revenue.

**Proposition 2.** Any optimal mechanism of the price-controlling data broker induces \(\varphi_G(c)\)-quasi-perfect price discrimination for \(G\)-almost all \(c \in C\). In particular, the optimal revenue is

\[
R^* = \int_C \left( \int_{\{v \geq \varphi_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc) - \bar{\pi}.
\]
According to Theorem 1 and Lemma 1, the optimal revenue of the data broker must also be $R^*$. This means that the additional ability to control prices does not benefit the data broker at all. In fact, as stated by Theorem 7 below, this ability is entirely irrelevant in terms of market outcomes.

**Theorem 7 (Outcome-Equivalence).** Exclusive retail, price-controlling data brokershipe and data brokershipe are outcome-equivalent.

In other words, Theorem 7 means that even though the data broker only affects the product market indirectly by selling consumer data, the market outcomes he induces are the same as those when he has more control over the product market by either becoming a price-controlling data broker or an exclusive retailer. More specifically, from the data broker’s perspective, having control over how the product is sold in addition to consumer data adds no extra values to his revenue. As for the producer, her profit in face of a data broker is the same as if she sells the product, as well as the exclusive right to sell the product, to this data broker. Preserving the access to consumers and the right to sell the product is in fact not more profitable. In addition, the allocation of the product induced by a data broker is the same as that induced by an exclusive retailer. Therefore, the channel through which the product is sold to the consumers does not affect the amount of products being produced, nor does it affect to whom the product is sold.

This outcome-equivalence result has several further implications. First, it implies that there are no incentives for the data broker to become more active, as the data broker’s revenue would remain the same even if he becomes a price-controlling data broker or an exclusive retailer. Second, from a policymaker’s perspective, it means that no further concerns should be raised even if a data broker eventually becomes more active. After all, the market outcomes and the amount of deadweight loss would remain the same. Overall, Theorem 7 provides a way to gauge how powerful the ability to design and sell market segmentations is: According to Theorem 7, this ability is so powerful that being able to further contract on outcomes in the product market provide no additional values to the data broker.

As another remark, the fact that the price-controlling data broker’s optimal revenue $R^*$ is an upper bound for the data broker’s optimal revenue completes the intuition behind the proof of Theorem 1 without the additional assumption (4) imposed in Section 4.2. To see this, since the price-controlling data broker’s optimal mechanisms always induce $\varphi_G$-quasi-perfect price discrimination for (almost) all $c \in C$ according to Proposition 2, proving Theorem 1 is essentially reduced to finding an incentive feasible $\varphi_G$-quasi-perfect mechanism. Furthermore, by the definition of $\varphi_G$, $c \leq \varphi_G(c) \leq \bar{p}_0(c)$ for all $c \in C$, and hence $\varphi_G$ satisfies the condition required by Lemma 3. As a result, combining Lemma 2 and Lemma 3, there is indeed an incentive feasible $\varphi_G$-quasi-perfect mechanism, which, by definition, generates revenue $R^*$, and hence is optimal. As noted at the end of the previous section, some of
the individual rationality constraints are binding under this optimal mechanism. In fact, without (4), the reason that the price controlling data broker’s revenue $R^*$ is the correct upper bound, as opposed to $R$ constructed in Section 4, is because some of the individual rationality constraints are binding under price-controlling data broker’s optimal mechanism (see more discussions in Section 7).

6 Extensions: Consumers’ Private Information

Given the amount of consumer data that can be collected, their predictive power is approaching perfect estimations of consumers’ values. Nonetheless, it is still imperative to explore the economic implications of the possibility when the consumers have some private information. This section extends the baseline model in Section 3 and allows the consumers to retain some pieces of information.

To model this, let $\Theta$ be a finite set that denotes the consumer characteristics that can be disclosed by the data broker. Suppose that among the consumers, their characteristics $\theta \in \Theta$ are distributed according to $\beta_0 \in \Delta(\Theta)$. These characteristics are informative about the consumers’ values but there may still be variation in values even among the consumers who share the same characteristics. Specifically, given any $\theta \in \Theta$, suppose that among the consumers who share characteristic $\theta$, their values are distributed according to $m^\theta \in \Delta(V)$ and $m^\theta$ induces a demand $D_\theta \in \mathcal{D}$ (i.e., $D_\theta(p) := m^\theta([p, \bar{v}])$ for all $p \in P$) for each $\theta \in \Theta$. Moreover, suppose that $\{\text{supp}(D_\theta)\}_{\theta \in \Theta}$ forms a partition of $V$ and supp($D_\theta$) is an interval for all $\theta \in \Theta$. A market segmentation in this environment is defined by $s \in \Delta(\Delta(\Theta))$ such that

$$\int_{\Delta(\Theta)} \beta(\theta)s(d\beta) = \beta_0(\theta),$$

for all $\theta \in \Theta$.

In other words, the available consumer characteristics is only partially informative about the consumers’ values in a way that any particular characteristic can only identify which interval a particular consumer’s value belongs to. As a result, even when $\theta$ is perfectly revealed, the producer would still be unable to identify each consumer’s value. For any $p \in V$, let

$$D_0(p) := \sum_{\theta \in \Theta} D_\theta(p)\beta_0(\theta).$$

$D_0 \in \mathcal{D}$ then describes the market demand in this environment. Moreover, a market segmentation $s$ induces market segments $\{D_\beta\}_{\beta \in \text{supp}(s)}$ and

$$\sum_{\beta \in \text{supp}(s)} D_\beta(p)s(\beta) = D_0(p),$$

for all $p \in V$, where $D_\beta(p) := \sum_{\theta \in \Theta} D_\theta(p)\beta(\theta)$ for any $\beta \in \Delta(\Theta)$ and any $p \in V$. 
When the consumers’ values can never be fully disclosed, it is clear that their surplus will increase. After all, it is no longer possible for the producer to charge the consumers their values as the additional variation in values given by $D_\theta$ always allows some consumers to buy the product at a price that is below their values. Nevertheless, as shown in Theorem 8, under any optimal mechanism, consumer surplus must be lower than the case when all the information about $\theta$ is revealed to the producer. That is, the main implication of Theorem 3—for the consumers, the presence of a data broker is no better than a scenario where their data is fully revealed to the producer—is still valid even when the consumers retain some private information.

**Theorem 8.** For any $(\{D_\theta\}_{\theta \in \Theta}, \beta_0)$ and for any cost distribution $G$, an optimal mechanism always exists. Furthermore, the consumer surplus under any optimal mechanism of the data broker is lower than the case when $\theta$ is fully disclosed.

The intuition behind Theorem 8 is simple. Since there are only finitely many characteristics, identifying the consumers’ characteristic $\theta$ effectively enables the producer to categorize the consumers into finitely many “blocks” so that every possible value belongs to one and only one block. As a result, when changing prices within each block of values, the trading volume is only affected by purchasing decisions of the consumers whose values are within that block. Such separability allows the data broker can always construct a mechanism that increases its revenue if the consumer surplus is higher than that when the characteristic $\theta$ is not full-revealed.\textsuperscript{22}

In addition to the surplus extraction result, the characterization of the optimal mechanisms can be generalized as well. That is, with proper regularity conditions, there is an optimal mechanism that is analogous to the canonical $\varphi_G$-quasi-perfect mechanism introduced in Section 4. To state this result, given any $(\{D_\theta\}_{\theta \in \Theta}, \beta_0)$, for each $\theta \in \Theta$, write $\text{supp}(D_\theta)$ as $[l(\theta), u(\theta)]$. For any $p \in V$, let $\theta_p \in \Theta$ be the unique $\theta$ such that $p \in (l(\theta), u(\theta)]$. For any $c \in C$, let $\hat{p}_0(c)$ be the largest optimal price for the producer with marginal cost $c \in C$ under the demand whose support contains $\bar{p}_0(c)$.\textsuperscript{23} Also, let $\hat{\varphi}_G(c) := \min\{\varphi_G(c), \hat{p}_0(c)\}$ for all $c \in C$. Furthermore, given any function $\psi : C \to \mathbb{R}_+$, say that a mechanism $(\sigma, \tau)$ is a canonical $\psi$-quasi-perfect segmentation if the producer with marginal cost $c \in C$ when reporting truthfully, receives $\bar{\pi}$, and if for any $c \in C$, and for any $\beta \in \text{supp}(\sigma(c))$, either

$$\beta(\theta') = \beta_{\psi(c)}(\theta') := \begin{cases} \beta_0(\theta'), & \text{if } u(\theta') < \psi(c) \text{ and } u(\theta) \geq \psi(c) \\ \sum_{\theta' : \hat{u}(\theta') \geq \psi(c)} \beta_0(\theta'), & \text{if } u(\theta') \geq \psi(c) \text{ and } \theta' = \theta \\ 0, & \text{otherwise} \end{cases},$$

\textsuperscript{22}A more detailed argument can be found in the proof, which is provided in the Online Appendix

\textsuperscript{23}That is, $\hat{p}_0(c) := \bar{p}_0(c)$. Notice that $\hat{p}_0(c) \leq \bar{p}_0(c)$ for all $c \in C$. Moreover, in the case where the data broker can disclose all the information about the value $v$, $\hat{p}_0(c) = \bar{p}_0(c)$ for all $c \in C$.  
for any $\theta, \theta' \in \Theta$, or

$$\text{supp}(\beta) = \{\theta' : l(\theta') \leq \psi(c)\} \cup \{\theta\} \quad (14)$$

for some $\theta \in \Theta$ with $l(\theta) \geq \psi(c)$ and

$$\beta(\theta') = \beta_0(\theta'). \quad (15)$$

for all $\theta' \in \Theta$ such that $u(\theta') < \psi(c)$.

With these definitions, Theorem 9 below prescribes an optimal mechanism for the data broker.

**Theorem 9.** For any $(\{D_\theta\}_{\theta \in \Theta}, \beta_0)$ and any distribution of marginal cost $G$ such that the function $c \mapsto \max\{\phi_G(c) - \hat{p}_0(c), 0\}$ is nondecreasing and that $D_0$ is regular, there is a canonical $\hat{\varphi}_G$-quasi-perfect mechanism that is optimal.

### 7 Discussions

#### 7.1 Sufficient Conditions and Relaxations of Assumption 1

As noted in Section 4, Assumption 1 has a sufficient condition (4). To better understand (4), recall that $\phi_G(c)$ is the actual marginal cost $c$ plus the information rent $G(c)/g(c)$. Meanwhile, $\hat{p}_0(c)$ can be written as $\hat{p}_0(c) = c + \xi_0(c)$, where $\xi_0(c) := \hat{p}_0(c) - c$ is the *monopoly mark-up* that the producer charges under uniform pricing. From this perspective, (4) is equivalent to $G(c)/g(c) \leq \xi_0(c)$, for all $c \in C$. That is, the information rent that the producer retains due to asymmetric information about her marginal cost is less than her monopoly mark-up. Furthermore, as (4) means that the optimal uniform price must be greater than the virtual cost, it can also be interpreted as the gains from trade being large enough.

Although the results introduced above rely on Assumption 1, the main purpose of Assumption 1 is to ensure that as a revenue upper bound, the price-controlling data broker’s problem has a closed form solution. After all, by Lemma 4, the price-controlling data broker’s problem is essentially a nonlinear screening problem with one-dimensional allocation space and type-dependent outside options. A common feature of such problems is that the characterization of the optimal mechanisms involves Lagrange multipliers in general (see, for instance, Lewis and Sappington (1989) and Jullien (2000)). Assumption 1, however, yields a closed form solution for the price-controlling data broker’s problem (Proposition 2), which in turn allows an explicit construction of an incentive feasible mechanism for the data broker that attains the revenue upper bound. Consequently, many of the results can be extended

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24A formal argument can be found in an earlier version of this paper (Yang, 2020c), where gains from trade are measured by a demand shifter than moves the market demand to the right on the real line.
to environments without Assumption 1, including the main characterization, the surplus extraction result and the associated implications.\footnote{In an earlier version of this paper (Yang, 2020c), I provide a generalized version of Theorem 1 when $D_0$ is continuous. Specifically, I show that there exists a nondecreasing function $\varphi^*$ (may not necessarily be of a closed form) such that every optimal mechanism must be a $\varphi^*$-quasi-perfect mechanism. Furthermore, I proved a strengthened version of Theorem 3, which does not rely on any assumptions about $D_0$ and $G$ and ensures both the existence of an optimal mechanism, as well as the fact that any optimal mechanism must yield zero consumer surplus.}

7.2 Creating Market Segmentations by Consumer Data

Throughout the paper, a market segmentation is defined as a probability measure $s \in S$ that splits the market demand $D_0$ into several segments $D \in D$. Although this formalization of market segmentations is well-aligned with the literature on price discrimination, a more practical way to describe a market segmentation—especially in environments where segmentations are generated by consumer data—is to define a market segment as a subset of consumers identified by their characteristics that are correlated with their values of a product.

Clearly, when there are not enough of available underlying characteristics, there would not be many ways to split the market demand. For instance, in the motivating example in Section 1, if the only available characteristic is the resident-type, then the market demand can only be split in the way described by Figure 2. This suggests that for the data broker to be able to design any market segmentation, it is implicitly required that the underlying characteristics are “rich enough” (i.e., the data broker has a large enough dataset). In a companion note (Yang, 2020a), I formalize this observation and provide a precise definition of “richness”. While how the data broker should sell consumer data when there is only a limited set of available characteristics remains an open question, this formal connection guarantees that the data broker can generate any market segmentation $s \in S$ by partitioning the underlying characteristic space provided that it is rich enough. From this perspective, the results in this paper can be regarded as what the data broker can possibly achieve when he has an access to unlimited consumer data.

7.3 Source of Asymmetric Information

The results in previous sections are derived under an information structure where the producer has private information about her marginal cost. Although this informational assumption captures some of the features in retail markets, it apparently does not capture all of them. Specifically, one salient informational asymmetry between a data broker and a producer in the real world is that producers often know more about how consumers’ characteristics are related to their values for a particular product—perhaps due to their industry-specific knowl-
edge that is too costly for the data broker to acquire. While optimal selling mechanisms for the data broker under this general informational environment remain an open question, the methodology developed in this paper can still provide some insights. In particular, under a parameterized information structure where the producer has private information about the market condition (as opposed to her marginal cost), all the results derived in this paper continue to hold.

More specifically, consider the following alternative information structure. There is a unit mass of consumers with unit demand for a single product. Each consumer has value $v - \xi$, where $v \in [\underline{v}, \bar{v}] = V \subseteq \mathbb{R}_+$ is heterogeneous across consumers and are distributed according to $m^0 \in \Delta(V)$, while $\xi \in [0, \bar{v}]$ is the same across consumers. Both the consumers and the producer (with a commonly known marginal cost that is normalized to zero) know $\xi$, while the data broker only knows that $\xi$ is drawn from a distribution $G$. The interpretation is that the producer knows more about the market condition (i.e., a “demand shifter” described by $\xi$) than the data broker does. Meanwhile, market segmentations are defined as before: A market segmentation is a probability measure $s \in S \subseteq \Delta(D)$. It then follows that the demand in a market segment $D \in D$ with market condition $\xi$ is given by $D(p + \xi)$ (i.e., $D(p + \xi)$ is the share of consumers in segment $D$ who are willing to buy the product at price $p$). Under this setting, given a demand shifter $\xi$, under any market segment $D \in D$, the producer’s pricing problem is given by

$$\max_{p \geq 0} p D(p + \xi),$$

which, by letting $p' = p + \xi$, is equivalent to

$$\max_{p' \geq 0} (p' - \xi) D(p') = \pi(p, D, \xi).$$

As a result, the informational setting above where the producer privately knows a demand shifter is equivalent to the original model where the producer has a private marginal cost $\xi$, and hence all the results derived above continue to hold in this alternative setting.

### 7.4 Policy Implications

The results above have several broader policy implications. First, in terms of welfare, although Theorem 3 implies that data brokership is undesirable for the consumers, Theorem 4 shows that the total surplus is always higher with the presence of a data broker compared with an environment where no information about the consumers’ values can be disclosed. As a result, the answer to the question of whether a data broker is beneficial must depend on the objective of the policymaker and the kinds of redistributional policy tools available. If the policymaker’s objective is simply maximizing total surplus, or if redistributional tools
such as lump-sum transfers are available, then it is indeed beneficial to allow a data broker to sell consumer data. In contrast, if the policymaker also concerns themselves with consumer surplus, and if no effective redistributional policies are available, then the presence of a data broker can be extremely unfavorable. Therefore, regarding the policy debates about whether a data broker should be allowed to collect, use and trade consumer data, it is imperative to first identify the available redistributional tools and the relative importance among consumer surplus, producer profit and total surplus.

In the case where the policymaker does wish to improve consumer surplus and no effective redistributional policies are available, Theorem 5 prescribes a potential way to improve welfare. According to Theorem 5, if the data broker has to purchase the data from the consumers, and if the purchase takes place before the consumers learn their values, then data brokership would be Pareto-improving compared with uniform pricing. As a result, if the policymaker can establish the consumers’ property right for their own data, as well as a channel for the data broker to compensate the consumers, then not only the consumers can secure their surplus as if their data is not used for price discrimination (via compensation), but also the entire economy can benefit from data brokership, because less deadweight loss will be generated.

Finally, regardless of the policymaker’s objective, as long as it depends only on the market outcomes, the discussions in Section 5.3 facilitates the evaluation of whether a certain market regime is desirable than another. According to Theorem 6, if the policymaker is able to eliminate the asymmetric information regarding the production cost, integrating the data broker with the producer can be beneficial. This result is due to the fact that even when the data broker only sells consumer data to the producer, the consumer surplus is still zero. Consequently, revealing the producer’s private marginal cost and encouraging vertical integration are beneficial as it does not affect the consumer surplus but eliminates all the informational frictions.

Meanwhile, the equivalence result given by Theorem 7 implies that as long as it is the producer who bears the production cost, however active the data broker is in the product market does not affect market outcomes at all. This means that, on the one hand, the data broker has no incentive to become even more active in the product market rather than only selling consumer data. In fact, together with other potential costs that are abstracted away from the model (e.g., inventory costs, shipping costs and other transaction costs), participating directly in product market can be less profitable than merely selling consumer data to the producer. On the other hand, even if the data broker does become more active in

\[26\text{For instance, just as what is stipulated by the recent regulation of the European Union, General Data Protection Regulation (GDPR, Art. 7), consumers’ property right for their own data can be better protected by prohibiting all the processing of personal data unless the data subject has consented the use.}\]
the product market, it still raises no further concerns to the policymaker. Thus, any policy intervention that prohibits the data broker entering the product market by either gaining control over prices (e.g., by establishing an online platform and allows the producer to trade on this platform while controlling the prices) or obtaining the exclusive right to (re)-sell the product would be unnecessary. However, the other side of this result is that even if the data broker is not active in the product market at all, the policymaker should be equally concerned as if the data broker is very active in the product market.

8 Conclusion

In sum, this paper studies a scenario where a data broker sells consumer data and creates market segmentations. In this paper, I characterize the optimal mechanisms of the data broker and conclude that consumer surplus is always zero, that data brokership generates more total surplus than uniform pricing, and that the ability to control prices in the product market is irrelevant. Several extensions are also considered, including the case in which consumers possess some private information that cannot be disclosed and the environment where targeted marketing is available.

Several aspects can become future study topics. First, although one of the extensions of this paper considers a scenario where targeted marketing is possible, it abstracts away from the possibility that a data broker can generate “match values” between the producers and consumers. By assuming that every group of consumers can buy every product as long as they see it, the matching aspect between consumers and producers is omitted. After all, there is effectively no competition among the producers when there is no “scarcity” of consumers. Furthermore, the consumers’ characteristics that govern the match values can also be their private information. Second, although one of the extensions consider the case where the consumers can preserve some private information, it is restricted to certain environments. A natural direction of future research is to explore the data broker’s optimal mechanisms and their implications in a setting where the feasible market segmentation is restricted by a Blackwell upper bound. Lastly, the producer is assumed to be a product monopoly in this paper. It would be economically relevant to explore the consequences of consumer-data brokership under different industrial structures.
Appendix

A  Details of $\mathcal{D}$

Below I first discuss more formally about the properties of the set $\mathcal{D}$. Recall that $\mathcal{D} = \mathcal{D}([v, \bar{v}])$ is the collection of nonincreasing and left-continuous functions $D$ on $[v, \bar{v}]$ such that $D(v) = 1$ and $D(\bar{v}^+) = 0$. Since for every $D \in \mathcal{D}$, there exists a unique probability measure $m^D \in \Delta(V)$ such that $D(p) = m^D(\{v \geq p\})$ for all $p \in V$, I define the topology on $\mathcal{D}$ by the following notion of convergence: For any $\{D_n\} \subseteq \mathcal{D}$ and any $D \in \mathcal{D}$, $\{D_n\} \to D$ if and only if for any bounded continuous function $f : V \to \mathbb{R}$,

$$\lim_{n \to \infty} \int_V f(v)m^{D_n}(dv) = \int_V f(v)m^D(dv).$$

This would correspond to the weak-* topology on $\Delta(V)$ and hence this topology on $\mathcal{D}$ is also called the weak-* topology. As a result, $\mathcal{D}$ is a Polish space. Furthermore, notice that under this topology, $\{D_n\} \to D$ if and only if $\{D_n(p)\} \to D(p)$ for all $p \in V$ at which $D$ is continuous. Finally, for any $D \in \mathcal{D}$, let $S_D$ denote the collection of $s \in \Delta(D)$ such that (1) holds with $D_0$ being replaced by $D$ (so that $S_{D_0} = S$). Also, let $D^{-1}$ denote the inverse demand of $D$, where $D^{-1}$ is defined as

$$D^{-1}(q) := \sup\{p \in V : D(p) \geq q\}, \forall q \in [0, 1].$$

B  Proofs for the Main Results

B.1  Crucial Properties of Quasi-Perfect Schemes

This section summarizes some crucial properties of quasi-perfect segmentation schemes. The proofs of these properties are mostly technical and are not directly related to the arguments of the proofs of main results, and therefore are relegated to the Online Appendix.

Lemma 5. Consider any nondecreasing function $\psi : C \to \mathbb{R}_+$ with $c \leq \psi(c)$ for all $c \in C$. Suppose that for any $c \in C$, $\sigma(c) \in S$ is a $\psi(c)$-quasi-perfect segmentation for $c$. Then,

1. $\int_{\mathcal{D}} D(p)\sigma(dD|c) = D_0(p)$ for all $p \in V$ and for all $c \in C$.

2. $\sigma : C \to \Delta(D)$ is measurable.

3. $\int_{\mathcal{D}} D(\bar{D}_D(c))\sigma(dD|c) = D_0(\psi(c))$ for all $c \in C$.

4. $\int_{\mathcal{D}} \bar{D}_D(c)D(\bar{D}_D(c))\sigma(dD|c) = \int_{\{v \geq \psi(c)\}} vD_0(dv)$ for all $c \in C$.

B.2  Proof of Proposition 2

In this section, I first prove Proposition 2 and obtain an upper bound for the data broker’s revenue. That is, I first solve the relaxed problem where the prices are also contractable. To this end, it would be useful to introduce the revenue-equivalence formula for the price-controlling data broker.
Lemma 6. For the price-controlling data broker, a mechanism $(\sigma, \tau, \gamma)$ is incentive compatible if and only if

1. There exists some $\bar{\tau} \in \mathbb{R}$ such that for any $c \in C$,
   \[
   \tau(c) = \int_{D} \int_{\mathbb{R}_+} (p - c)D(p)\gamma(dp|D,c)\sigma(dD|c) - \int_{c} \int_{D} \int_{\mathbb{R}_+} D(p)\gamma(dp|D,z)\sigma(dD|z) \, dz - \bar{\tau}. \]

2. The function $c \mapsto \int_{D} \int_{\mathbb{R}_+} D(p)\gamma(dp|D,c)\sigma(dD|c)$ is nonincreasing.

The proof of Lemma 6 follows directly from the standard envelope arguments and therefore is omitted. In addition to Lemma 6, since both price and market segmentations can be contracted by the price-controlling data broker, and since the producer’s private information is one dimensional, the price controlling data broker’s problem can effectively be summarized by a one dimensional screening problem where the data broker contracts on quantity at which a perfectly-price discriminating producer sells. This is summarized by Lemma 7 below.

Lemma 7. There exists an incentive feasible mechanism that maximizes the price-controlling data broker’s revenue. Furthermore, the price-controlling data broker’s revenue maximization problem is equivalent to the following

\[
\sup_{q \in \mathcal{Q}} \int_{C} \left( \int_{0}^{q(c)} \left( D_0^{-1}(q) - \phi_G(c) \right) dq \right) G(dc) - \bar{\pi} \quad (17)
\]

s.t. $\bar{\pi} + \int_{c}^{\bar{\tau}} q(z) \, dz \geq \bar{\pi} + \int_{c}^{\bar{\tau}} D_0(p_0(z)) \, dz.$

where $\mathcal{Q}$ is the collection of nonincreasing functions that map from $C$ to $[0,1]$.

The proof of Lemma 7 can be found in the Online Appendix. Essentially, the argument is to summarize $\sigma$ and $\gamma$ by

\[
q(c) = \int_{D \times \mathbb{R}_+} D(p)\gamma(dp|D,c)\sigma(dD|c)
\]

for all $c$. As the producer’s private information is one dimensional, it turns out that it is sufficient for the price-controlling data broker to design quantity $q$. By the revenue equivalence formula (Lemma 6), the objective function of (17) equals to the broker’s expected revenue; the monotonicity condition $q \in \mathcal{Q}$ corresponds to global incentive compatibility constraints; and the inequality constraints in (17) are equivalent to the individual rationality constraints.

With Lemma 7, the price-controlling data broker’s revenue maximization problem can be solved explicitly.

Proof of Proposition 2. Let $R^*$ be the value of (17) and consider the dual problem of (17). By weak duality, it suffices to find a Borel measure $\mu^*$ and a feasible $q^* \in \mathcal{Q}$ such that $q^*$ is a solution of

\[
\sup_{q \in \mathcal{Q}} \left[ \int_{C} \left( \int_{0}^{q(c)} \left( D_0^{-1}(q) - \phi_G(c) \right) dq \right) G(dc) - \bar{\pi} + \int_{C} \left( \int_{c}^{\bar{\tau}} (q(z) - D_0(p_0(z))) \, dz \right) \mu^*(dc) \right] \quad (18)
\]
and that
\[ \int_C \left( \int_{\mathbb{C}} (q^*(z) - D_0(\overline{\mathcal{P}_0}(z))) \, dz \right) \mu^*(dc) = 0. \]  
\hspace{1cm} \tag{19}

To this end, define \( M^* : C \to [0, 1] \) as the following
\[ M^*(c) := \lim_{z \downarrow c} g(z)(\phi_G(z) - \overline{\mathcal{P}_0}(z))^+, \quad \forall c \in C. \]  
\hspace{1cm} \tag{20}

By definition, \( M^* \) is right-continuous. Also, by Assumption 1, \( M^* \) is nondecreasing and hence \( M^* \) a CDF. Let \( \mu^* \) be the Borel measure induced by \( M^* \). Notice that \( \text{supp}(\mu^*) = [c^*, \overline{c}] \), where \( c^* := \inf\{c \in C : \phi_G(c) > \overline{\mathcal{P}_0}(c)\} \).

For any \( q \in \mathcal{Q} \), by interchanging the order of integrals and then rearranging, (18) can be written as
\[ \sup_{q \in \mathcal{Q}} \left[ \int_C \left( \int_0^q (D_0^{-1}(q) - \overline{\mathcal{P}_0}(c)) \, dq \right) \mu^*(dc) \right], \]  
\hspace{1cm} \tag{21}

where \( \overline{\phi}_G := \min\{\phi_G, \overline{\mathcal{P}_0}\} \).

To solve (21), let \( \varphi_G \) be the ironed virtual cost. That is, \( \varphi_G \) is defined by the following procedure:

Let \( h : [0, 1] \to \mathbb{R}_+ \) be defined as \( h(q) := \phi_G(G^{-1}(q)) \), and define \( H : [0, 1] \to \mathbb{R}_+ \), \( K : [0, 1] \to \mathbb{R}_+ \) as
\[ H(q) := \int_0^q h(s) \, ds \quad \text{and} \quad K := \text{conv}(H). \]

Then, for every \( q \in [0, 1] \) let \( k(q) := K'(q) \) and define \( \varphi_G \) as \( \varphi_G(c) := k(G(c)) \). Also, let \( \overline{\varphi}_G := \min\{\varphi_G, \overline{\mathcal{P}_0}\} \).

Now notice that for any \( q \in \mathcal{Q} \), and for any \( c \in C \),
\[ \int_0^q (D_0^{-1}(q) - \overline{\mathcal{P}_0}(c)) \, dq = \int_0^q (D_0^{-1}(q) - \overline{\varphi}_G(c)) \, dq + (\overline{\varphi}_G(c) - \overline{\mathcal{P}_0}(c))q(c). \]  
\hspace{1cm} \tag{22}

Moreover, using integration by parts, since \( K(0) = H(0) \) and \( K(1) = H(1) \),
\[ \int_C (\overline{\varphi}_G(c) - \overline{\mathcal{P}_0}(c))q(c)G(dc) = \int_C (\varphi_G(c) - \phi_G(c))q(c)G(dc) = - \int_C (K(G(c)) - H(G(c)))q(dc) \leq 0, \]  
\hspace{1cm} \tag{23}

where the first equality follows from the observation that \( \overline{\phi}_G(c) = \overline{\varphi}_G(c) = \phi_G(c) = \varphi_G(c) = \overline{\mathcal{P}_0}(c) \) for all \( c \geq c^* \), which is due to Assumption 1, and the inequality follows from the fact that \( K = \text{conv}(H) \) and that \( q \) is nonincreasing for any \( q \in \mathcal{Q} \).

Meanwhile, notice that for any \( q \in \mathcal{Q} \),
\[ \int_C \left( \int_0^q (D_0^{-1}(q) - \overline{\varphi}_G(c)) \, dq \right) G(dc) \leq \int_C \left( \int_0^{D_0(\overline{\varphi}_G(c))} (D_0^{-1}(q) - \overline{\varphi}_G(c)) \, dq \right) G(dc), \quad \forall q \in \mathcal{Q}. \]  
\hspace{1cm} \tag{24}

In addition, since \( \overline{\varphi}_G(c) = \overline{\mathcal{P}_0}(c) \) for all \( c \in (c^*, \overline{c}] \) and since \( K(G(c)) < H(G(c)) \) on an interval \([c_1, c_2]\) if and only if \( \varphi_G \) is a constant on that interval, which implies that \( D_0 \circ \overline{\varphi}_G \) is a constant on that interval, it must be that
\[ \int_C (\overline{\varphi}(c) - \overline{\mathcal{P}_0}(c))q^{+\varphi}(c)G(dc) = - \int_C (K(G(c)) - H(G(c)))D_0(\overline{\varphi}(c))(dc) = 0. \]  
\hspace{1cm} \tag{25}

Together with (22), and (23), (24) for any \( q \in \mathcal{Q} \),
\[ \int_C \left( \int_0^q (D_0^{-1}(q) - \overline{\varphi}_G(c)) \, dq \right) G(dc) \leq \int_C \left( \int_0^{D_0(\overline{\varphi}_G(c))} (D_0^{-1}(q) - \overline{\varphi}_G(c)) \, dq \right) G(dc). \]
Also, since \( \overline{\varphi}_G \) is nondecreasing by definition, \( D_0 \circ \overline{\varphi}_G \) is indeed a solution of \( (21) \) and hence a solution of \( (18) \).

Moreover, since \( \overline{\varphi}_G \leq \overline{p}_0 \), for all \( c \in C \), \( \int_C \overline{D}_0(\overline{\varphi}_G(z)) \, dz \geq \int_C \overline{D}_0(\overline{p}_0(z)) \, dz \). Therefore, \( D_0 \circ \overline{\varphi}_G \in Q \) is feasible choice in the primal problem \( (17) \). Meanwhile, since \( M^*(c) = 0 \) for all \( c \in [c, c^*] \) and since \( \overline{\varphi}_G(c) = \overline{p}_0(c) \) for all \( c \in (c^*, \overline{c}] \), the complementary slackness condition \( (19) \) is also satisfied. Together, \( D_0 \circ \overline{\varphi}_G \) is indeed a solution of \( (17) \). Finally, by definition of \( D_0^{-1} \), it then follows that

\[
R^* = \int_C \left( \int_0^{D_0(\overline{\varphi}_G(c))} (D_0^{-1}(q) - \phi_G(c)) \, dq \right) G(dc) - \overline{\pi} = \int_C \left( \int_{\{v \geq \overline{\varphi}_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc) - \overline{\pi}.
\]

The see that any solution of the price-controlling data broker’s problem must induce \( \overline{\varphi}_G(c) \)-quasi-perfect price discrimination for \( G \) almost all \( c \in C \), consider any optimal mechanism \( (\sigma, \tau, \gamma) \) of the price-controlling data broker. By optimality, it must be that \( \mathbb{E}_G[\tau(c)] = R^* \) and that the indirect utility of the producer with marginal cost \( \overline{c} \) is \( \overline{\pi} \). Thus, by Lemma 7, it must be that

\[
\int_C \left( \int_D \left( \int_{\mathbb{R}^+} (p - \phi_G(c)) \sigma(dD[c]) \right) \sigma(dD[c]) \right) G(dc) = \int_C \left( \int_{\{v \geq \overline{\varphi}_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc),
\]

which is equivalent to

\[
\int_C \left( \int_{D \times \mathbb{R}^+} (p - \overline{\varphi}_G(c)) D(p) \gamma(dp|D, c) \sigma(dD[c]) \right) G(dc) + \int_C (\overline{\varphi}_G(c) - \phi_G(c)) \mathbf{D}_\gamma^c(c) G(dc) \]

\[
= \int_C \left( \int_{\{v \geq \overline{\varphi}_G(c)\}} (v - \overline{\varphi}_G(c)) D_0(dv) \right) G(dc) + \int_C (\overline{\varphi}_G(c) - \phi_G(c)) D_0(\overline{\varphi}_G(c)) G(dc),
\]

where \( \mathbf{D}_\gamma^c(c) := \int_{D \times \mathbb{R}^+} D(p) \gamma(dp|D, c) \sigma(dD[c]) \) for all \( c \in C \). Moreover, since for any \( c \in C \),

\[
\int_{D \times \mathbb{R}^+} (p - \overline{\varphi}_G(c)) D(p) \gamma(dp|D, c) \sigma(dD[c]) \leq \int_D \max[(p - \overline{\varphi}_G(c))D(p)\sigma(dD[c]) \leq \int_V (v - \overline{\varphi}_G(c))^+ D_0(dv),
\]

it must be that

\[
\int_C \left( \overline{\varphi}_G(c) - \phi_G(c) \right) \mathbf{D}_\gamma^c(c) G(dc) \geq \int_C \left( \overline{\varphi}_G(c) - \phi_G(c) \right) D_0(\overline{\varphi}_G(c)) G(dc).
\]

Meanwhile, since \( (\sigma, \tau, \gamma) \) is incentive compatible, Lemma 6 implies that \( \mathbf{D}_\gamma^c \) is nonincreasing in \( c \). Together with \( (23) \) and \( (25) \), we have

\[
\int_C \left( \overline{\varphi}_G(c) - \phi_G(c) \right) \mathbf{D}_\gamma^c(c) G(dc) \geq \int_C \left( \overline{\varphi}_G(c) - \phi_G(c) \right) D_0(\overline{\varphi}_G(c)) G(dc).
\]

Furthermore, since \( \overline{\varphi}_G(c) = \overline{p}_0(c) \leq \phi_G(c) \) for all \( c \in (c^*, \overline{c}] \) and \( \overline{\varphi}_G(c) = \phi_G(c) \), by the definition of \( M^* \) given by \( (20) \), and by using integration by parts, \( (29) \) is equivalent to

\[
\int_C \left( \int_z \left( \mathbf{D}_\gamma^c(z) - D_0(\overline{p}_0(z)) \right) \, dz \right) M^*(dc) \leq 0
\]
Meanwhile, since \((\sigma, \tau, \gamma)\) is individually rational, for any \(c \in C\),
\[
\int_c^\pi \left( D_\gamma'(z) - D_0(p_D(z)) \right) \, dz \geq 0.
\]
Thus, (30) must hold with equality, which in turn implies that (29) must hold with equality. Together with (27), (28) must hold with equality for \(G\)-almost all \(c \in C\), \((\sigma(c), \gamma(c))\). That is, \((\sigma, \gamma)\) must induce \(\varphi_G(c)\)-quasi-perfect price discrimination for \(G\)-almost all \(c \in C\). This completes the proof. \(\blacksquare\)

B.3 Proof of Lemma 1

Proof of Lemma 1. For necessity, consider any incentive compatible mechanism \((\sigma, \tau)\). First notice that, by Proposition 1 of Yang (2020b), \(\pi_D : C \to \mathbb{R}_+\) is convex and absolutely continuous on \(C\) for any \(D \in \mathcal{D}\) with \(\pi'_D(c) = -D(p_D(c))\) for all \(p \in P\) and for almost all \(c \in C\). Moreover, since for any \(D \in \mathcal{D}\) and for any \(p \in P\) \(|\pi'_D(c)| = |D(p_D(c))|\) \(\leq 1\), for almost all \(c \in C\), the order of integral and differential can be interchanged. That is, for any \(c, c' \in C\),
\[
\frac{d}{dc} \int_D \pi_D(c)\sigma(dD|c') = \int_D \pi'_D(c)\sigma(dD|c') = -\int_D (p_D(c))\sigma(dD|c'). \tag{31}
\]
As such, for any \(c' \in C\), the function \(c \mapsto \int_D \pi_D(c)\sigma(dD|c')\) is convex and, by (31), has an almost-everywhere derivative \(-\int_D D(p_D(c))\sigma(dD|c'),\) for any \(p \in P\). Now let \(u(c, c') := \int_D \pi_D(c)\sigma(dD|c') - \tau(c')\) for all \(c, c' \in C\) be a producer’s profit if her report is \(c'\) and marginal cost is \(c\). By the Lebesgue dominated convergence theorem, \(u(\cdot, c')\) is convex and absolutely continuous on \(C\) for all \(c' \in C\) as \(\pi_D\) is convex and absolutely continuous for all \(D \in \mathcal{D}\). Furthermore, since the mechanism \((\sigma, \tau)\) is incentive compatible, by the envelope theorem (Milgrom and Segal, 2002), let \(U(c) := u(c, c')\), we then have
\[
U(c) = U(\pi) - \int_c^\pi \frac{\partial}{\partial c} u(z, z) \, dz = U(\pi) + \int_c^\pi \left( \int_D D(p_D(z))\sigma(dD|z) \right) \, dz. \tag{32}
\]
Assertion 1 then follows after rearranging.

Furthermore, for any mechanism \((\sigma, \tau)\) satisfying assertion 1 (and hence (32)) with any \(p \in P\), we have
\[
U(c) - u(c, c') = (U(c) - U(c')) + \int_D (\pi_D(c) - \pi_D(c'))\sigma(dD|c')
\[
= \int_c^{c'} \left( \int_D D(p_D(z))\sigma(dD|z) - \int_D D(p_D(z))\sigma(dD|c') \right) \, dz
\[
= \int_c^{c'} \left( \int_D D(p_D(z))(\sigma(dD|z) - \sigma(dD|c')) \right) \, dz,
\]
where the first equality follows from the fundamental theorem of calculus, and the second equality follows from (31). Therefore, for any mechanism \((\sigma, \tau)\) satisfying assertion 1 with any \(p \in P\), \(U(c) \geq u(c, c')\) for all \(c, c' \in C\) if and only if assertion 2 holds. This completes the proof. \(\blacksquare\)

B.4 Proof of Lemma 2

Proof of Lemma 2. Given any nondecreasing function \(\psi : C \to \mathbb{R}_+\), and any \(\psi\)-quasi-perfect scheme \(\sigma : C \to \mathcal{S}\), suppose that for any \(c \in C\), \(\psi(z) \leq p_D(z)\), for Lebesgue almost all \(z \in [c, c]\) and for all \(D \in \text{supp}(\sigma(c))\).
Then, for any \( c, c' \in C \) with \( c < c' \),
\[
\int_c^{c'} \left( \int_D D(\mathfrak{p}_D(z)) \sigma(dD|z) - \sigma(dD|c') \right) \, dz = \int_c^{c'} \left( D_0(\psi(z)) - \int_D D(\mathfrak{p}_D(z)) \sigma(dD|c') \right) \, dz \\
\geq \int_c^{c'} \left( D_0(\psi(z)) - \int_D D(\psi(z)) \sigma(dD|c') \right) \, dz \\
= \int_c^{c'} (D_0(\psi(z)) - D_0(\psi(z))) \, dz \\
= 0,
\]
where the first equality follows from \((8)\), the inequality follows from \((10)\), and the second equality follows from \(\sigma(z) \in S\) for all \( z \in [c, c'] \). Meanwhile, for any \( c, c' \in C \) with \( c > c' \),
\[
\int_c^{c'} \left( \int_D D(\mathfrak{p}_D(z)) \sigma(dD|c) - \sigma(dD|c') \right) \, dz = \int_c^{c'} \left( \int_D D(\mathfrak{p}_D(z)) \sigma(dD|c) - D_0(\psi(z)) \right) \, dz \\
= \int_c^{c'} (\min\{D_0(c), D_0(z)\} - D_0(\psi(z))) \, dz \\
\geq 0,
\]
where the first equality again follows from \((8)\), and the second equality follows from the fact that \( c < c' \) and from the definition of quasi-perfect segmentations.\(^{27}\) Therefore, by Lemma 1, there exists a transfer \( \tau \) such that \((\sigma, \tau)\) is incentive compatible, as desired. \(\blacksquare\)

### B.5 Proof of Theorem 1

**Proof of Theorem 1.** I first show that the data broker’s optimal revenue must be the same as the price-controlling data broker’s optimal revenue \( R^* \). To see this, since \( c \leq \varphi_G(c) \leq \mathfrak{p}_0(c) \) for all \( c \in C \) and \( \varphi_G \in \mathbb{R}^C_+ \) is nondecreasing, by Lemma 3, there exists a \( \varphi_G \)-quasi-perfect scheme \( \sigma^* \in S^C \) that satisfies \((10)\).

Together with Lemma 2, there exists a transfer \( \tau^* \) such that \((\sigma^*, \tau^*)\) is incentive compatible. Moreover, since \( \sigma \in S^C \) is a \( \varphi_G \)-quasi-perfect scheme, by assertion 3 and assertion 4 of Lemma 5, for any \( c \in C \),
\[
\int_D (\mathfrak{p}_D(c) - \phi_G(c)) \, D(\mathfrak{p}_D(c)) \sigma^*(dD|c) = \int_{\{v \geq \varphi_G(c)\}} (v - \phi_G(c)) \, D_0(dv).
\]

Also, by possibly adding a constant to the transfer \( \tau^* \), the indirect utility of the producer with cost \( \tau \), \( U(\tau) \), equals to \( \bar{\pi} \) under the mechanism \((\sigma^*, \tau^*)\). Thus, for any \( c \in C \),
\[
\int_D \pi_D(c) \sigma^*(dD|c) - \tau(c) = U(\bar{\pi}) + \int_c^{\pi} \left( \int_D D(\mathfrak{p}_D(z)) \sigma^*(dD|z) \right) \, dz \\
= \bar{\pi} + \int_c^{\pi} D_0(\varphi_G(z)) \, dz \\
\geq \bar{\pi} + \int_c^{\pi} D_0(\mathfrak{p}_0(z)) \, dz \\
= \pi_0(c),
\]

\(^{27}\)More specifically, for any \( c \in C \), since \( \sigma(c) \) is a \( \psi(c) \)-quasi-perfect segmentation for \( c \), for any \( z > c \) and for any \( D \in \text{supp}(\sigma(c)) \), if \( D(c) > 0 \) and \( \max(\text{supp}(D)) \geq z \), then \( \mathfrak{p}_D(z) = \mathfrak{p}_D(c) \); if \( D(c) > 0 \) and \( \max(\text{supp}(D)) < z \), then \( D(\mathfrak{p}_D(z)) = 0 \); while if \( D(c) = 0 \) then \( D(z) = 0 \).
where the first equality follows from Lemma 1, the second equality follows from assertion 3 of Lemma 5, the inequality follows from $\overline{\varphi}_G \leq \overline{p}_0$ and the last equality follows from (5). As a result, $(\sigma^*, \tau^*)$ is individually rational and, together with (33) and Lemma 1,

$$
\mathbb{E}[\tau^*(c)] = \int_C \left( \int_D (\overline{p}_D(c) - \phi_G(c)) D(\overline{p}_D(c)) \sigma^*(dD|c) \right) G(dc) - \overline{\pi} = \int_C \left( \int_{\{v \geq \overline{\varphi}_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc) - \overline{\pi} = R^*,
$$
as desired.

Since the data broker’s optimal revenue is $R^*$ and since (33) holds for any $\overline{\varphi}_G$-quasi-perfect scheme $\sigma$, by Lemma 1, any incentive feasible $\overline{\varphi}_G$-quasi-perfect mechanism must give revenue $R^*$ and hence is optimal.

Conversely, to see why any optimal mechanism must be a $\overline{\varphi}_G$-quasi-perfect mechanism, consider any optimal mechanism $(\sigma, \tau)$. As it is optimal and incentive compatible, by Lemma 1,

$$
R^* = \mathbb{E}[\tau(c)] = \int_C \left( \int_D (p_D(c) - \phi_G(c)) D(p_D(c)) \sigma(dD|c) \right) G(dc) - \overline{\pi},
$$

for any $p \in P$. Also, since $(\sigma, \tau)$ is incentive compatible, for any $p \in P$, the function $D^*_p : C \to [0,1]$, defined as

$$
D^*_p(c) := \int_D D(p_D(c)) \sigma(dD|c), \forall c \in C
$$
is nonincreasing.\(^{28}\) Thus, by (23),

$$
\int_C \overline{\phi}_G(c) \left( \int_D D(p_D(c)) \sigma(dD|c) \right) G(dc) \geq \int_C \overline{\varphi}_G(c) \left( \int_D D(p_D(c)) \sigma(dD|c) \right) G(dc).
$$

Moreover, since $(\sigma, \tau)$ is individually rational, by Lemma 1, it must be that

$$
\int_{\tau(c)} \left( \int_D D(p_D(z)) \sigma(dD|z) \right) dz \geq \int_{c} \overline{\varphi}_G(c) D_0(\overline{p}_0(z)) dz, \forall c \in C.
$$

Furthermore, since $\sigma^*$ is a $\overline{\varphi}_G$-quasi-perfect scheme, Lemma 5 implies that, for all $c \in C$,

$$
D^*_p(\sigma^*)(c) = \int_D D(\overline{p}_D(c)) \sigma^*(dD|c) = D_0(\overline{\varphi}_G(c)).
$$

Together with (25), we have

$$
\int_C \overline{\varphi}_G(c) D_0(\overline{\varphi}_G(c)) G(dc) = \int_C \overline{\phi}_G(c) D_0(\overline{\varphi}_G(c)) G(dc).
$$

Now suppose that $(\sigma, \tau)$ is not a $\overline{\varphi}_G$-quasi-perfect mechanism or it does not induce $\overline{\varphi}_G(c)$-quasi-perfect price discrimination for a positive $G$-measure of $c$, then there exists $p \in P$, a positive $G$-measure of $c$ and a positive $\sigma(c)$-measure of $D \in D$ such that either $p_D(c) < \overline{p}_D(c)$, or $D(c) > 0$ and either $\#\{v \in \text{supp}(D) :$

\(^{28}\)To see this, notice that $U$ is convex since it is a pointwise supremum of convex functions, which is because $\pi_D(c)$ is convex for all $D$. Lemma 1 implies that the derivative of $U$ is $-D^*_p$ and thus $D^*_p$ must be nonincreasing.
\(v \geq \overline{\varphi}_G(c)\} \neq 1\) or \(\max(\text{supp}(D)) \notin P_D(c)\), which imply that there is a positive \(G\)-measure of \(c\) and a positive \(\sigma(c)\)-measure of \(D\) such that

\[
\int_{\{v \geq \overline{\varphi}_G(c)\}} (v - \overline{\varphi}_G(c)) D(dv) \geq \int_{\{v \geq \overline{p}_D(c)\}} (v - \overline{\varphi}_G(c)) D(dv)
= (p_D(c) - \overline{\varphi}_G(c)) D(p_D(c)) + \int_{\{v \geq p_D(c)\}} (v - p_D(c)) D(dv)
\geq (p_D(c) - \overline{\varphi}_G(c)) D(p_D(c)),
\]

with at least one inequality being strict. Therefore,

\[
\int_C \left( \int_D (p_D(c) - \overline{\varphi}_G(c)) D(p_D(c)) \sigma(dD|c) \right) G(dc) < \int_C \left( \int_V (v - \overline{\varphi}_G(c))^+ D_0(dv) \right) G(dc).
\] (38)

Moreover, by (34), since

\[
\int_C \left( \int_D (p_D(c) - \overline{\varphi}_G(c)) D(p_D(c)) \sigma(dD|c) \right) G(dc) + \int_C (\overline{\varphi}_G(c) - \phi_G(c)) \left( \int_D D(p_D(c)) \sigma(dD|c) \right) G(dc)
= \int_D \left( \int_C (p_D(c) - \phi_G(c)) D(p_D(c)) \sigma(dD|c) \right) G(dc)
= \int_C \left( \int_{\{v \geq \overline{\varphi}_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc)
= \int_C \left( \int_V (v - \overline{\varphi}_G(c))^+ D_0(dv) \right) G(dc) + \int_C (\overline{\varphi}_G(c) - \phi_G(c)) D_0(\overline{\varphi}_G(c)) G(dc),
\]

together with, (38), it must be that

\[
\int_C (\varphi_G(c) - \phi_G(c)) \left( \int_D D(p_D(c)) \sigma(dD|c) \right) G(dc) \geq \int_C (\overline{\varphi}_G(c) - \phi_G(c)) \left( \int_D D(p_D(c)) \sigma(dD|c) \right) G(dc)
> \int_C (\overline{\varphi}_G(c) - \phi_G(c)) D_0(\overline{\varphi}_G(c)) G(dc)
= \int_C (\overline{\varphi}_G(c) - \phi_G(c)) D_0(\overline{\varphi}_G(c)) G(dc),
\]

where the first inequality follows from (35) and the equality follows from (37). Furthermore, since \(\overline{\varphi}_G(c) = \phi_G(c)\) for all \(c \in [\underline{c}, \overline{c}]\) and \(\overline{\varphi}_G(c) = \overline{\varphi}_G(c) = \overline{\varphi}_0(c)\) for all \(c \in (\overline{c}, \overline{c}]\), it then follows that

\[
\int_{\underline{c}}^{\overline{c}} (\phi_G(c) - \overline{\varphi}_0(c)) \left( \int_D D(p_D(c)) \sigma(dD|c) \right) G(dc) < \int_{\underline{c}}^{\overline{c}} (\phi_G(c) - \overline{\varphi}_0(c)) D_0(\overline{\varphi}_0(c)) G(dc),
\]

Using integration by parts, this is equivalent to

\[
\int_{\underline{c}}^{\overline{c}} \left( \int_{\underline{c}}^{\overline{c}} D(p_D(z)) \sigma(dD|z) dz \right) M^*(dc) < \int_{\underline{c}}^{\overline{c}} \left( \int_{\underline{c}}^{\overline{c}} D_0(\overline{\varphi}_0(z)) dz \right) M^*(dc),
\]

where \(M^*\) is defined in (20). However, by (36) and by the fact that \(M^*\) is a CDF of a Borel measure, which is due to Assumption 1,

\[
\int_{\underline{c}}^{\overline{c}} \left( \int_{\underline{c}}^{\overline{c}} D(p_D(z)) \sigma(dD|z) dz \right) M^*(dc) \geq \int_{\underline{c}}^{\overline{c}} \left( \int_{\underline{c}}^{\overline{c}} D_0(\overline{\varphi}_0(z)) dz \right) M^*(dc),
\]

a contradiction. Therefore, \(\sigma\) must be a \(\overline{\varphi}_G\)-quasi-perfect scheme and must induce \(\overline{\varphi}_G(c)\)-quasi-perfect price discrimination for \(G\)-almost all \(c \in C\). Together with Lemma 1, and the fact that \(U(\overline{\varphi}) = \overline{\varphi}\) under any optimal mechanism, \((\sigma, \tau)\) must be a \(\overline{\varphi}_G\)-quasi-perfect mechanism. This completes the proof. \(\blacksquare\)
B.6 Proof of Theorem 2

Proof of Theorem 2. By the proof of Lemma 3 in the main text. When \( D_0 \) is regular, since \( c \leq \varphi_G(c) \leq \bar{p}_0(c) \) for all \( c \in C \), the canonical \( \varphi_G \)-quasi-perfect scheme \( \sigma^* \) defined in (3) is implementable. Therefore, there exists \( \tau^* \) such that \( (\sigma^*, \tau^*) \) is an incentive feasible \( \varphi_G \)-quasi-perfect mechanism. By Theorem 1, \( (\sigma^*, \tau^*) \) is optimal. ■

C Proofs of Other Main Results

C.1 Proof of Theorem 3

Proof of Theorem 3. Let \( (\sigma, \tau) \) be any optimal mechanism. By Theorem 1, \( (\sigma, \tau) \) must be a \( \varphi_G \)-quasi-perfect mechanism and induces \( \varphi_G \)-quasi-perfect price discrimination. Therefore, for \( G \)-almost all \( c \in C \) and for \( \sigma(c) \)-almost all \( D \in \mathcal{D} \), \( D(p) = 0 \) for any \( p > \bar{p}_D(c) = \max(\text{supp}(D)) \) and thus consumer surplus under \( (\sigma, \tau) \) is

\[
\int_C \left( \int_{\{c \geq \bar{p}_D(c)\}} (v - \bar{p}_D(c)) D_0(dv) \right) \sigma^*(dD|c) G(dc) = \int_C \left( \int_{\{c \geq \bar{p}_0(c)\}} (v - \bar{p}_0(c)) D_0(dv) \right) G(dc) = 0,
\]

where the first equality follows from integration by parts. ■

C.2 Proof of Proposition 1

Proof of Proposition 1. Since \( P_0(c) \) is a singleton for (Lebesgue)-almost all \( c \in C \) and since \( G \) is absolutely continuous, consumer surplus under uniform pricing does not depend which selection \( p \in P \) is used. Therefore, by Theorem 1, the difference between the data broker’s optimal revenue and the consumer surplus under uniform pricing is

\[
\int_C \left( \int_{\{c \geq \varphi_G(c)\}} (v - \phi_G(c)) D_0(dv) \right) G(dc) - \bar{\pi} - \int_C \left( \int_{\{c \geq \bar{p}_0(c)\}} (v - \bar{p}_0(c)) D_0(dv) \right) G(dc) = \int_C \left( \int_c D_0(\bar{p}_0(z)) dz - \frac{G(c) - g(c)}{g(c)} D_0(\bar{p}_0(c)) \right) G(dc)
\]

where the second equality follows from Lemma 1, the first inequality follows from the fact that \( \varphi_G(c) < \bar{p}_0(c) \) if and only if \( \varphi_G(c) < \bar{p}_0(c) \), and the last inequality follows from (23) and (25). This completes the proof. ■
C.3 Proof of Theorem 7

Proof of Theorem 7. By Lemma 4, whose proof can be found in the Online Appendix, it suffices to prove outcome-equivalence between data brokership and price-controlling data brokership. By Proposition 2 and Theorem 1, both the exclusive retailer and the data broker have optimal revenue $R^*$. Furthermore, for any optimal mechanism $(\sigma, \tau)$ of the data broker and any optimal mechanism $(\hat{\sigma}, \hat{\tau}, \hat{\gamma})$ of the price-controlling data broker, both of them must induce $\varphi_G(c)$-quasi-perfect price discrimination for $G$-almost all $c \in C$. In particular, for $G$-almost all $c \in C$, all the consumers with $v \geq \varphi_G(c)$ buys the product by paying their values and all the consumers with $v < \varphi_G(c)$ do not buy the product. Thus, the consumer surplus and the allocation of the product induced by $(\sigma, \tau)$ and $(\hat{\sigma}, \hat{\tau}, \hat{\gamma})$ are the same.

In addition, for the data broker’s optimal mechanism $(\sigma, \tau)$, Theorem 1 implies that $\sigma$ must be a $\varphi_G$-quasi-perfect scheme and hence by (7) and (8) and Lemma 1, for Lebesgue almost all $c \in C$,

$$\int_D \sigma_D(c) \sigma(dD|c) - \tau(c) = \bar{\pi} + \int_c^\infty \left( \int_D D(\bar{p}_D(z)) \sigma(dD|z) \right) dz = \bar{\pi} + \int_c^\infty D_0(\varphi_G(z)) dz. \quad (39)$$

Meanwhile, for the price-controlling data broker’s optimal mechanism $(\hat{\sigma}, \hat{\tau}, \hat{\gamma})$, since, by Proposition 1, it induces $\varphi_G(c)$-quasi-perfect price discrimination for almost all $c \in C$, it must be that $D_{\hat{\gamma}}(c) = D_0(\varphi_G(c))$. Together with Lemma 6, for any $c \in C$,

$$\int_D \left( \int_{\mathbb{R}_+} (p - c) D(p) \hat{\gamma}(dp|D,c) \right) \hat{\sigma}(dD|c) - \hat{\tau}(c) = \bar{\pi} + \int_c^\infty D_{\hat{\gamma}}(z) dz = \bar{\pi} + \int_c^\infty D_0(\varphi_G(z)) dz. \quad (40)$$

Thus, the producer’s profit under both $(\sigma, \tau)$ and $(\hat{\sigma}, \hat{\tau}, \hat{\gamma})$ are the same. This completes the proof. \hfill \Box

References


