Efficient Market Structures
Under Incomplete Information*

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Abstract

In economies with incomplete information, laissez-faire price competition is not, in
general, constrained Pareto efficient. But which market structures are? We consider an
environment in which firms have private information about costs and consumers make
discrete choices over goods. Surveying an expansive class of market structures, we
show that the constrained efficient ones are equivalent to price competition, but with
lump-sum transfers and yardstick price ceilings that depend on the prices of competing
firms.

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1 Introduction

It is well established that the acclaimed welfare theorems of Arrow (1951) and Debreu (1951) fail to hold in economies with incomplete information. Greenwald and Stiglitz (1986), for example, show that competitive equilibria in settings with incomplete information are generally not even constrained Pareto efficient. There exist government taxes or subsidies that can make everyone better off. Similarly, Guesnerie and Roberts (1984) show that quantity controls can generate Pareto improvements in second best worlds.\footnote{Incomplete information goes hand in hand with incomplete markets, and so, analogous arguments against constrained optimality in settings with missing markets have been made as well. See, for example, Hart (1975), McManus (1984), and Geanakoplos and Polemarchakis (1985).}

But existing results on the failure of the welfare theorems are generally derived under prespecified market structures.\footnote{A market structure describes the nature of competition between participating firms (McNulty, 1968; Baumol, 1982). Core elements of a market structure include the ease of entry and exit, the number of active sellers, the distribution of market shares, and firm conduct (i.e., sellers’ available competitive strategies and how those strategies translate into market equilibrium outcomes).} Regularly, the market structure is assumed to be Walrasian in the sense of Arrow (1951) and Debreu (1951), where every agent is a price taker. Within the scope of a Walrasian market structure, authors typically then show that there exists a different allocation that Pareto dominates the competitive equilibrium outcome. But it remains an open question whether a market structure other than perfect competition can further enhance welfare under incomplete information; and if so, what that market structure looks like.

This paper answers that question. Doing so requires making assumptions about the nature of the incomplete information and consumer preferences. We study an environment in which firms have independent and one-dimensional private information about costs and people choose one among many differentiated goods to consume.\footnote{Discrete choice representations of consumer demand like this have been commonplace in the modeling of product markets since McFadden (1973).} Under these assumptions, we then search across a large set of possible market structures to find the constrained Pareto efficient one. We prove that every constrained efficient market structure is equivalent to price competition, but with lump-sum transfers and a certain form of price controls—namely, firm-specific price ceilings that depend on the prices of competitors (i.e., “yardstick” price caps).

It is noteworthy that our definition of market structures includes a broad range of strate-
gies that firms might employ when competing. This expansive treatment accommodates the fact that styles of competition can differ substantially between markets. As Berry et al. (2019) note, “prices are determined in the food distribution industry via second price auction, in health care via bilateral bargaining, and in retail as posted prices.” We demonstrate that the class of market structures we search over nests each of these examples and more. Among others are pure monopoly, competing on price à la Bertrand (1883), competing on quantity à la Cournot (1838), competing over differentiated goods (Perloff and Salop, 1985), and consumer search (Varian, 1980; Narasimhan, 1988; Armstrong and Vickers, 2019).

The intuition behind our main characterization is as follows: Because we presume independent private types across firms, we can adopt the Myersonian approach to keep track of revenues as functions of an allocation of goods to consumers. Doing so implies that the efficient allocation must allocate goods to consumers who have the highest ex-post virtual surplus on the intensive margin (i.e., the difference between a consumer’s value for a good and the virtual marginal cost of the firm supplying the good). At the same time, the efficient allocation must also select the set of firms that generate the highest ex-ante virtual surplus on the extensive margin when taking fixed costs into account. Price competition implements the efficient allocation on the intensive margin. By properly designing lump-sum transfers between consumers and firms as functions of firms’ prices, one can incentivize firms to post prices that exactly reflect their virtual marginal cost. With virtual marginal costs (and, hence, firms’ private types) being reflected in prices, one can then select the most efficient firms based solely on the information collected from posted prices on the extensive margin. This selection criterion leads to the firm-specific price ceilings.

In contrast to virtually all existing work in welfare economics that pre-selects a market structure, we start with as bare an architecture as we can imagine. We assume general distributions of consumer tastes and firm costs. The number of participating firms is endogenous. Regarding the market structure, we only take as given the existence of a functioning market—an institution “for the consummation of transactions” (Stigler, 1957). Our approach is closer to the mechanism design literature that considers all possible interacting channels through which equilibrium outcomes can be determined. In particular, our model can be thought of as a generalization of Baron and Myerson (1982) that considers an economy with
more than one firm. With multiple firms, the allocation space becomes substantially richer. In our setting, the relevant allocation is the entire distribution of matches between consumers and goods, instead of a one-dimensional demand quantity.\footnote{See Amador and Bagwell (2021) and Guo and Shmaya (2019) for other recent papers related to Baron and Myerson (1982).}

The rest of this paper is organized as follows: In Section 2, we introduce the model, define market structures, and specify the welfare criterion for efficiency. Section 3 states the main result, and Section 4 has the proof. Section 5 provides an example of an efficient market structure. Section 6 concludes.

2 Model

2.1 Primitives

A number $N \geq 1$ of firms produce $N$ heterogeneous goods. Each firm $i$ has cost function $C_i(q) = \theta_i (q + \kappa_i)$, where $q$ is quantity and $\kappa_i \geq 0$ is commonly known. Meanwhile, $\theta_i \geq 0$ represents the firm’s cost efficiency and is private information. A lower $\theta_i$ implies that a firm is more cost-efficient. We assume that $\theta = (\theta_i)_{i=1}^N \in \mathbb{R}^N$ is independent and $\theta_i$ follows a distribution $G_i$, which has a support $\Theta_i := [\underline{\theta}_i, \overline{\theta}_i]$, with $0 \leq \underline{\theta}_i \leq \overline{\theta}_i < \infty$.

A unit mass of consumers stand ready to purchase. Each consumer has unit demand and heterogeneous values $v \in V \subseteq \mathbb{R}_+^N$, so that a consumer with value vector $v = (v_1, \ldots, v_N)$ has value $v_i$ for firm $i$’s good. The consumers’ values are distributed according to measure $F \in \Delta(V)$.

2.2 Market Structure

A market structure $\mathcal{M}$ is a tuple $\mathcal{M} = (S_i, r_i, \mu_i, t_i)_{i=1}^N$ that assigns firms’ strategies to (i) market entry probabilities, (ii) an allocation of goods to consumers, and (iii) firm revenues; where, for all $i$, $S_i$ is an arbitrary (measurable) set, $r_i$ is a mapping from $S := \prod_{i=1}^N S_i$ to $[0, 1]$, $t_i$ is a mapping from $S$ to $\mathbb{R}$, and $\mu_i$ is a mapping from $V \times S$ to $[0, 1]$, such that $\sum_{i=1}^N \mu_i(v|s) \leq 1$ for all $(v, s) \in V \times S$.

For any market structure, $S_i$ describes firm $i$’s available strategies (e.g., chosen price,
chosen quantity, or entry decision). Given any strategy profile \( s \in S = \prod_{i=1}^{N} S_i \), \( r_i(s) \in [0, 1] \) denotes the probability that firm \( i \) enters the market; \( \mu_i(v|s) \in [0, 1] \) represents the share of consumers with value \( v \) who receives firm \( i \)'s good, conditional on firm \( i \) being in the market; and \( t_i(s) \) is the revenue of firm \( i \). We normalize the firms’ outside options to zero, and we require that any market structure must allow an opt-out option \( s_0 \in S_i \) such that \( t_i(s_0, s_{-i}) = \mu_i(v|s_0, s_{-i}) = r_i(s_0, s_{-i}) = 0 \) for all \( i \), for all \( s_{-i} \in S_{-i} \), and for all \( v \in V \).

Given a market structure \( \mathcal{M} \), the timing of events is as follows: (1) types \( \{\theta_i\}_{i=1}^{N} \) are drawn independently from \( \{G_i\}_{i=1}^{N} \), and each firm privately observes its own type; (2) firms simultaneously choose \( s_i \) from \( S_i \); and (3) each firm \( i \) receives ex-post payoff

\[
\pi_i(s, \theta_i|\mathcal{M}) := t_i(s) - r_i(s)\theta_i \left( \int V \mu_i(v|s)F(dv) + \kappa_i \right).
\]

Notice that a market structure \( \mathcal{M} \) defines a Bayesian game where each firm \( i \) has private type \( \theta_i \in \Theta_i \), strategy space \( S_i \), and payoff function \( \pi_i(s, \theta_i|\mathcal{M}) \).

### 2.3 Examples of Market Structures

Here we provide some examples of market structures, in particular, monopoly, price competition, quantity competition, entry deterrence, search, and auctions.

**Example 1 (Monopoly).** The following market structure describes a monopoly market where firm \( i = 1 \) is a monopolist. Under this market structure, only firm 1 operates in the market and chooses a monopoly price \( s_1 \geq 0 \) at which to sell to consumers. Given price \( s_1 \), all consumers with value \( v_1 \geq s_1 \) buys from firm 1, whereas all other consumers do not buy.

Specifically, the strategy spaces are \( S_i = \mathbb{R}_+ \) for all \( i \); entry probabilities are \( r_1(s_1) = 1 \) and \( r_j(s_j) = 0 \) for all \( j \neq 1 \); revenues are \( t_i(s) := s_i \int V \mu_i(v|s)F(dv) \) for all \( i \); and the allocation of goods to consumers is

\[
\mu_i(v|s) = \begin{cases} 
1, & \text{if } i = 1 \text{ and } v_i \geq s_i \\
0, & \text{otherwise}
\end{cases}
\]

for all \( i \) and for all \( s \in S \).
Example 2 (Price Competition). The following market structure describes a price competition model. Under this market structure, all \(N\) firms operate in the market and compete on the price margin (i.e., each firm \(i\) sets price \(s_i \geq 0\)). After seeing firms’ prices \(s = (s_1, \ldots, s_N)\), a consumer buys from the firm providing the highest surplus.

Specifically, each firm \(i\) has strategy space \(S_i = \mathbb{R}_+\). Under any strategy profile \(s \in S\), firm \(i\)’s entry probability is \(r_i(s) = 1\) and revenue is \(t_i(s) = s_i \int_v \mu_i(v|s) F(dv)\), where \(\mu_i\) is given by

\[
\mu_i(v|s) = \begin{cases} \frac{1}{|M(v,s)|}, & \text{if } v_i - s_i = \max_j \{v_j - s_j\} \text{ and } v_i \geq s_i \\ 0, & \text{otherwise} \end{cases}
\]

for all \(i \in \{1, \ldots, N\}\) and for all \(s \in S\), with \(M(v,s) := \arg\max_i \{v_i - s_i\}\).\(^5\)

Example 3 (Quantity Competition). Suppose that \(F\) is atomless. Then there exists a market structure that describes quantity competition of which the classical Cournot model (Cournot, 1838) is a special case. Under this market structure, each firm \(i\) chooses quantity \(s_i \in [0,1]\) it wishes to sell. Market prices (and, hence, the allocation of goods) are determined through a system of inverse demand functions \(\{p_i\}_{i=1}^N\). For any \(i\), \(\mu_i\) is defined so that firm \(i\) sells \(s_i\) units at price \(p_i(s)\) if \(\sum_j s_j \leq 1\), and sells \(\frac{s_i}{\sum_j s_j}\) units at price 0 if \(\sum_j s_j > 1\). This arrangement is strategically equivalent to a quantity competition game with inverse demand functions \(\{p_i\}_{i=1}^N\).\(^6\)

Example 4 (Entry Deterrence). The following market structure describes a model where an incumbent (firm 1) can use price to deter entry of potential entrants à la Von Stackelberg (1934) sequential competition. Let firm 1’s strategy space be \(S_1 = \mathbb{R}_+\), and let all other firms’ strategy spaces be a tuple that consists of entry decisions and prices as functions of firm 1’s price. The entry probability \(r_1\) of firm 1 is set to 1 regardless of the strategy profile, whereas entry probabilities of all other firms \(i \neq 1\) are given by their strategies. Lastly, revenues \(\{t_i\}_{i=1}^N\) and good allocations \(\{\mu_i\}_{i=1}^N\) are determined by price competition among

\(^5\)Notice that with different specifications of the value distribution \(F\), this market structure corresponds to various canonical competition models. In particular, by assuming that \(v\) is perfectly correlated (i.e., \(v_1 = \ldots, v_N = v\) with \(F\)-probability 1), we have the classical Bertrand competition model (Bertrand, 1883), but with private marginal costs; by assuming that \(v\) is independent, we have the model à la Perloff and Salop (1985); by assuming that \(N = 2\) and that \(v\) is perfectly negatively correlated (i.e., \(v_1 + v_2 = 1\) with \(F\)-probability 1), we have the Hotelling location model (Hotelling, 1929).

\(^6\)See more details in Online Appendix C.
all firms that enter.\footnote{See more details in Online Appendix C.}

**Example 5 (Promotional Sales and Consumer Search).** Consider the price competition market structure given by Example 2, except that $\mu_i$ becomes

$$
\mu_i(v|s) = \begin{cases} 
\gamma_i + \left( 1 - \sum_{j=1}^{N} \gamma_j \right) \frac{1\{s_i \in M(v,s)\}}{|M(v,s)|}, & \text{if } v_i \geq s_i \\
0, & \text{if } v_i < s_i
\end{cases}
$$

This market structure then describes a model with “captive consumers” and “shoppers,” where each firm $i$ has $\gamma_i \in [0,1]$ share of captive consumers who can only see its price, while the remaining consumers can visit all firms and see all firms’ prices.\footnote{Notice that if $v$ is perfectly correlated so that with $F$-probability 1, $v_1 = v_2 = \cdots = v_N$, this market structure describes the promotional sales model of Armstrong and Vickers (2019), which in turn nests the model of Varian (1980) and Narasimhan (1988).}

**Example 6 (Reverse Auction with Many Buyers).** The following market structure describes a type of reverse auction where firms bid their prices and those with the lowest bid win and sell their goods to all consumers with values above that bid. Pharmacy Benefit Managers (PBMs) are known to employ this kind of auction.\footnote{We thank Fiona Scott Morton for kindly pointing us to this example. See Garthwaite and Morton (2017) for more discussion on PBMs. In brief, the PBM holds an auction across drugs that treat a particular medical condition. The drugs of the winning manufacturers (the sellers) with the lowest bids are put on the PBM’s formulary for doctors to prescribe and patients (the consumers) to purchase.} Under this market structure, $S = \mathbb{R}_+^N$, $t_i(s) = s_i \int_V \mu_i(v|s)F(dv)$, $r_i(s) = 1$ for all $s \in S$ and for all $i$, and

$$
\mu_i(v|s) = \begin{cases} 
1\{s_i \in M(s)\} \frac{1}{|M(s)|}, & \text{if } s_i \leq v_i \\
0, & \text{if } s_i > v_i
\end{cases}
$$

for all $i$, for all $v \in V$, and for all $s \in S$, where $M(s) := \text{argmin}_i \{s_i\}$.

**Discussion.** As hinted by the many examples above, any Bayesian game that models competition among $K \leq N$ firms can be regarded as a market structure. As such, our analysis of market structures applies to all possible static models of competition with fixed preferences and technology, regardless of a model’s assumptions about firm conduct, market power, or price determination. Any dynamic model that can be represented in strategic form
is also eligible, as are markets in which prices are determined via bilateral bargaining.

Of course, not all competitive games among $K \leq N$ firms have an equilibrium. Furthermore, even if an equilibrium exists, some equilibria might be extremely difficult to characterize. A great benefit of our framework is that, as explained below, it bypasses explicit characterizations of equilibria and only focuses on the outcomes. Among this broad range of market structures and equilibria, our main interest is to characterize the efficient market structures and explore ways to implement them. To this end, we first formally define our notion of efficiency.

2.4 Defining Efficiency

For any market structure $\mathcal{M} = (S_i, r_i, \mu_i, t_i)_{i=1}^N$, and for any Bayes-Nash equilibrium $\sigma = \prod_{i=1}^N \sigma_i$ of the induced Bayesian game, where $\sigma_i : \Theta_i \to \Delta(S_i)$ is firm $i$’s equilibrium strategy, let

$$\Pi_i(\theta_i | \mathcal{M}, \sigma) := \mathbb{E}_{\theta_i \sim \mathcal{M}} \left[ \int_S \pi_i(s, \theta_i | \mathcal{M}) \sigma(ds | \theta) \right]$$

denote firm $i$’s interim profit, and let

$$\Sigma(\mathcal{M}, \sigma) := \mathbb{E}_{\theta} \left[ \int_{V \times S} \sum_{i=1}^N v_i \mu_i(v | s) \sigma(ds | \theta) F(dv) - \sum_{i=1}^N \int_S t_i(s) \sigma(ds | \theta) \right].$$

denote the expected consumer surplus. With this notation, we have the following definition of efficiency:

**Definition 1.** A market structure $\mathcal{M} = (S_i, r_i, \mu_i, t_i)_{i=1}^N$ is **efficient** if there exists a Bayes-Nash equilibrium $\sigma$ in the Bayesian game induced by $\mathcal{M}$, as well as a collection of nondecreasing, right-continuous functions $\{\Lambda_i\}_{i=1}^N$ on $\Theta_i$, with $0 \leq \Lambda_i(\theta_i) \leq G_i(\theta_i),$\(^{10}\) such that for

\(^{10}\)Notice that this definition nests that of Baron and Myerson (1982). Specifically, when $N = 1$ and when $\Lambda_i(\theta_i) = (1 - \alpha) G_i(\theta_i)$ for all $i$, for all $\theta_i$, and for some $\alpha \in [0, 1]$, efficient market structures reduce to that of Baron and Myerson (1982).
any market structure $\mathcal{M}'$ and any Bayes-Nash equilibrium $\sigma'$ of the induced game,

$$\Sigma(\mathcal{M}, \sigma) + \sum_{i=1}^{N} \int_{\Theta_i} \Pi(\theta_i|\mathcal{M}, \sigma) \Lambda_i(d\theta_i) \geq \Sigma(\mathcal{M}', \sigma') + \sum_{i=1}^{N} \int_{\Theta_i} \Pi(\theta_i|\mathcal{M}', \sigma') \Lambda_i(d\theta_i).$$

3 Efficiency of PRYCE CAP Market Structures

In what follows, we present our main result. As noted in the previous section, our main interest is in characterizing the efficient market structures and exploring practical ways to implement them. Although there are infinitely many possible market structures and some of them can be extremely complex, we show that the efficient market structures are “simple,” in the sense that any efficient market structure is equivalent to one that belongs to a natural class. This class of market structures involves price competition with lump-sum transfers and firm-specific price ceilings that depend on the chosen prices of competitors. We call these price ceilings yardstick price caps, and we refer to this class as PRYCE CAP market structures, which we define next.

**Definition 2.** $\mathcal{M} = (S_i, r_i, \mu_i, t_i)_{i=1}^{N}$ is a price competition market structure with lump-sum transfers and yardstick price caps (PRYCE CAP) if, for any $i$,

1. $S_i = \mathbb{R}_+$.  
2. For any $s \in S$, $r_i(s) = 1\{s_i \leq \bar{p}_i(s_{-i})\}$, for some $\bar{p}_i : S_{-i} \to \mathbb{R}_+ \cup \{\infty\}$.  
3. For any $v \in V$, and for any $s \in S$,

   $$\mu_i(v|s) = \begin{cases} 1, & \text{if } v_i - s_i > \max_{\{j|r_j(s) = 1, j \neq i\}}(v_j - s_j)^+ \text{ and } r_i(s) = 1 \\ 0, & \text{if } v_i - s_i < \max_{\{j|r_j(s) = 1, j \neq i\}}(v_j - s_j)^+ \text{ or } r_i(s) = 0 \end{cases}.$$  
4. For any $s \in S$, $t_i(s) = s_i \int_{V} \mu_i(v|s) F(dv) - \tau_i(s_i)$, for some $\tau_i : S_i \to \mathbb{R}$.

---

11As a remark, our definition of efficiency implies that we assign the same weight to each individual consumer. This welfare criterion follows that of Baron and Myerson (1982). A consequence is that even under an efficient market structure, an individual consumer might end up with negative surplus ex post from taxation, even if aggregate surplus is positive.

12In naming the price caps, we use the word “yardstick” in a way similar to Shleifer (1985)’s use of the word, in that a firm’s regulation depends on characteristics of other firms.
Under a PRYCE CAP market structure, each firm \( i \) simultaneously announces a price \( s_i \geq 0 \). Given the announced prices \( s = (s_1, \ldots, s_N) \), a firm is first selected into the market based on whether its announced price \( s_i \) is below its price cap \( \bar{p}_i(s_{-i}) \). The price caps and the rules for market entry are thus intimately linked. When choosing a price to announce, a firm accounts for both its own price cap and the effect that its choice will have on the price caps of other firms. Among the firms that enter the market, consumers then see the announced prices and decide which firm to buy from. Finally, each firm is compensated or taxed via lump-sum transfers from consumers. This transfer amount \( \tau_i(s_i) \) depends only on a firm’s own price.

Notice that if \( \bar{p}_i(s_{-i}) = \infty \) and \( \tau_i(s_i) = 0 \) for all \( i \) and for all \( s \), a PRYCE CAP market structure reduces to a pure price competition model (see Example 2 above). From this perspective, PRYCE CAP market structures can be regarded as generalizations of pure price competition models that are commonly assumed, with the differences being re-distributional transfers \( \{\tau_i\}_{i=1}^N \) and yardstick price caps \( \{\bar{p}_i\}_{i=1}^N \).

With the formal definition of PRYCE CAP market structures presented, we now state our main result.

**Theorem 1.** Any efficient market structure is equivalent to a PRYCE CAP market structure.

The significance of Theorem 1 is that, among infinitely many market structures, the efficient ones are equivalent to a PRYCE CAP market structure. This means that price competition, together with interventions solely in the form of lump-sum transfers and price ceilings, is enough to achieve constrained Pareto efficiency. PRYCE CAP market structures emerge as efficient out of an expansive set of market environments in which firms and consumers partake, with each environment potentially experiencing enormously complicated forms of firm conduct, barriers to entry, and regulatory policies.

Furthermore, Theorem 1 implies that omniscient knowledge about the market setting is not required to implement an efficient market structure. More precisely, implementing a PRYCE CAP market structure does not require knowledge about each individual consumer’s value vector \( \mathbf{v} \). With proper lump-sum transfers and yardstick price caps (which merely
depend on the *distribution* of consumer values and firm types), firms would post correct prices and consumers would sort themselves into the efficient allocation.

The fact that any efficient market structure is equivalent to a PRYCE CAP market structure sheds light on which regulatory policies are necessary and which are not. After all, Theorem 1 implies that if firms compete on price, any regulatory policy other than lump-sum transfers and price ceilings is unwarranted for reaching efficiency in an environment like ours. In other words, lump-sum transfers and price ceilings can be regarded as *minimal* regulatory policies.

### 4 Proof of Theorem 1

This section provides the proof of Theorem 1. First, notice that by the revelation principle (Myerson, 1979), it is without loss to restrict attention to incentive compatible and individually rational direct mechanisms. A direct mechanism is a market structure \((S, r, \mu, t)\) where \(S_i = \Theta_i\) for all \(i\). For simplicity, we refer to a direct mechanism as a mechanism, and we denote it by \((r, \mu, t)\) hereafter when there is no confusion.

A mechanism is said to be incentive compatible if, for all \(i\) and for all \(\theta_i, \theta'_i \in \Theta_i\),

\[
E_{\theta_{-i}} \left[ t_i(\theta_i, \theta_{-i}) - r_i(\theta_i, \theta_{-i})\theta_i \left( \int_V \mu_i(v|\theta_i, \theta_{-i})F(dv) + \kappa_i \right) \right] \\
\geq E_{\theta_{-i}} \left[ t_i(\theta'_i, \theta_{-i}) - r_i(\theta'_i, \theta_{-i})\theta_i \left( \int_V \mu_i(v|\theta'_i, \theta_{-i})F(dv) + \kappa_i \right) \right], \tag{IC}
\]

and it is said to be individually rational if, for all \(\theta_i \in \Theta_i\),

\[
E_{\theta_{-i}} \left[ t_i(\theta_i, \theta_{-i}) - r_i(\theta_i, \theta_{-i})\theta_i \left( \int_V \mu_i(v|\theta_i, \theta_{-i})F(dv) + \kappa_i \right) \right] \geq 0. \tag{IR}
\]

Under any incentive compatible and individually rational mechanism \((r, \mu, t)\), firm \(i\)'s interim expected profit is

\[
\Pi_i(\theta_i|r, \mu, t) = E_{\theta_{-i}} \left[ t_i(\theta_i) - r_i(\theta)\theta_i \left( \int_V \mu_i(v|\theta)F(dv) + \kappa_i \right) \right],
\]
while the expected consumer surplus is

\[ \Sigma(r, \mu, t) := \mathbb{E}_{\theta} \left[ \sum_{i=1}^{N} r_i(\theta) \int_{V} \mu_i(v) c_i F(dv) - \sum_{i=1}^{N} t_i(\theta) \right]. \]

As a result, an incentive compatible and individually rational mechanism is efficient if and only if it is the solution to the following problem:

\[
\sup_{(r, \mu, t)} \left[ \sum_{i=1}^{N} \int_{\Theta_i} \Pi_i(|r, \mu, t|) \Lambda_i(d\theta_i) + \Sigma(r, \mu, t) \right],
\]

subject to (IC) and (IR), for some collection of nondecreasing and right-continuous functions \( \{ \Lambda_i \} \) with \( 0 \leq \Lambda_i(\theta_i) \leq G_i(\theta_i) \) for all \( \theta_i \in \Theta_i \).

Meanwhile, using the standard envelope arguments, we can characterize incentive compatibility by a revenue equivalence formula and a monotonicity condition, as summarized by the following lemma.

**Lemma 1.** A mechanism \((r, \mu, t)\) is incentive compatible if and only if, for all \( i \), there exists a constant \( \bar{t}_i \in \mathbb{R} \) such that

1. For any \( i \) and for any \( \theta_i \in \Theta_i \),

\[
\mathbb{E}_{\theta_{-i}} \left[ t_i(\theta_i, \theta_{-i}) \right] = \bar{t}_i + \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta) \int_{V} \mu_i(v) F(dv) + \kappa_i \right] + \int_{\theta_i}^{\bar{\theta}_i} r_i(x, \theta_{-i}) \left( \int_{V} \mu_i(v|x, \theta_{-i}) F(dv) + \kappa_i \right) \, dx.
\]

2. For any \( i \), the function

\[
\theta_i \mapsto \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_{V} \mu_i(v|\theta_i, \theta_{-i}) F(dv) + \kappa_i \right) \right]
\]

is nonincreasing.
From Lemma 1, for any incentive compatible mechanism \((r, \mu, t)\), and for all \(i\),

\[
\mathbb{E}_\theta [t_i(\theta)] = \int_{\Theta_i} \mathbb{E}_{\theta_{-i}} [t_i(\theta_i)] G_i(d\theta_i)
\]

\[
= \bar{t}_i + \int_{\Theta_i} \theta_i \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_{V} \mu_i(v|\theta_i) F(dv) + \kappa_i \right) \right] G_i(d\theta_i)
\]

\[
+ \int_{\Theta_i} G_i(\theta_i) r_i(\theta_i, \theta_{-i}) \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_{V} \mu_i(v|\theta_i, \theta_{-i}) F(dv) + \kappa_i \right) \right] d\theta_i.
\]

Thus, expected consumer surplus can be written as

\[
\Sigma(r, \mu, t) = \mathbb{E}_\theta \left[ \sum_{i=1}^{N} r_i(\theta_i, \theta_{-i}) \int_{V} v_i \mu_i(v|\theta_i, \theta_{-i}) F(dv) \right]
\]

\[
- \sum_{i=1}^{N} \int_{\Theta_i} \theta_i \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_{V} \mu_i(v|\theta_i) F(dv) + \kappa_i \right) \right] G_i(d\theta_i)
\]

\[
- \sum_{i=1}^{N} \int_{\Theta_i} G_i(\theta_i) r_i(\theta_i, \theta_{-i}) \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_{V} \mu_i(v|\theta_i, \theta_{-i}) F(dv) + \kappa_i \right) \right] d\theta_i - \sum_{i=1}^{N} \bar{t}_i.
\]

Meanwhile, for each firm \(i\),

\[
\int_{\Theta_i} \Pi_i(\theta_i|r, \mu, t) \Lambda_i(d\theta_i) = \int_{\Theta_i} \Lambda_i(\theta_i) \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_{V} \mu_i(v|\theta_i, \theta_{-i}) F(dv) + \kappa_i \right) \right] d\theta_i + \Lambda_i(\bar{\theta}_i) \bar{t}_i
\]

With the above expressions, we now consider a relaxed problem of (1). To this end, we first introduce the following lemma summarizing the virtual cost functions \(\phi_i^{\Lambda_i}\).

**Lemma 2.** For any \(i\) and for any nondecreasing, right-continuous function \(\Lambda_i\) with \(0 \leq \Lambda_i(\theta_i) \leq G_i(\theta_i)\), there exists a nondecreasing function \(\phi_i^{\Lambda_i} : \Theta_i \rightarrow \mathbb{R}_+\) such that

\[
\int_{\Theta_i} \theta_i Q_i(\theta_i) G(d\theta_i) + \int_{\Theta_i} (G_i(\theta_i) - \Lambda_i(\theta_i)) Q_i(\theta_i) d\theta_i \leq \int_{\Theta_i} \phi_i^{\Lambda_i}(\theta_i) Q_i(\theta_i) G_i(d\theta_i)
\]

for any nonincreasing function \(Q_i : \Theta_i \rightarrow \mathbb{R}_+\), and the equality holds whenever \(Q_i\) is measurable with respect to the \(\sigma\)-algebra generated by \(\phi_i^{\Lambda_i}\).

**Proof.** This follows from Theorem 2 and Theorem 3 of Monteiro and Svaite (2010). \(\blacksquare\)

Combining Lemma 1 and Lemma 2, one can observe that the value of (1) is bounded
from above by the solution of

$$\sup_{r, \mu} \left\{ \mathbb{E}_\theta \left[ \sum_{i=1}^N r_i(\theta) \left( \int_V (v_i - \phi_i^\Lambda_i(\theta_i)) \mu_i(v|\theta) F(dv) - \phi_i^\Lambda_i(\theta_i) \kappa_i \right) \right] - \sum_{i=1}^N (1 - \Lambda_i(\theta_i)) \bar{t}_i \right\},$$

subject to

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_V \mu_i(v|\theta_i, \theta_{-i}) F(dv) + \kappa_i \right) \right]$$

is nonincreasing. (3)

Moreover, by Lemma 1, any individually rational mechanism must have $\bar{t}_i \geq 0$ for all $i$. Thus, it is without loss to set $\bar{t}_i = 0$ for all $i$.

In what follows, we characterize the solution of (1) by finding a solution to (2) first and then verifying that the objective of (1) equals the objective of (2) under this solution. To this end, define a market structure $(r^*, \mu^*, t^*)$ as follows: For any $\theta \in \Theta$, let $\mathcal{E}^*(\theta)$ be the largest solution (in terms of the strong set order) of

$$\max_{\mathcal{E} \subseteq \{1, \ldots, N\}} \left( \int_V \max_{i \in \mathcal{E}} (v_i - \phi_i^\Lambda_i(\theta_i))^+ F(dv) - \sum_{i \in \mathcal{E}} \phi_i^\Lambda_i(\theta_i) \kappa_i \right).$$

Then, let

$$\mu_i^*(v|\theta) := \begin{cases} \frac{1}{|\mathcal{M}^*(v, \theta)|}, & \text{if } v_i \geq \phi_i^\Lambda_i(\theta_i) \text{ and } i \in \mathcal{M}^*(v, \theta), \\ 0, & \text{otherwise} \end{cases},$$

where $\mathcal{M}^*(v, \theta) := \arg\max_{j \in \mathcal{E}^*(\theta)} \{v_j - \phi_j^\Lambda_j(\theta_j)\}$, for all $i$, for all $v \in V$, and for all $\theta \in \Theta$; and

$$r_i^*(\theta) = 1\{i \in \mathcal{E}^*(\theta)\}$$

for all $i$ and for all $\theta \in \Theta$; and

$$t_i^*(\theta) = T_i^*(\theta_i)$$

:= $\mathbb{E}_{\theta_{-i}} \left[ r_i^*(\theta_i) \left( \int_V \mu_i^*(v|\theta) F(dv) + \kappa_i \right) - \int_{\theta_i} r_i^*(x, \theta_{-i}) \left( \int_V \mu_i^*(v|x, \theta_{-i}) F(dv) + \kappa_i \right) dx \right],$

for all $i$ and for all $\theta \in \Theta$. 

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Lemma 3. The mechanism \((r^*, \mu^*, t^*)\) solves (2). Furthermore,

\[
\sum_{i=1}^{N} \int_{\Theta_i} \Pi(\theta_i|r^*, \mu^*, t^*) \Lambda_i(d\theta_i) + \Sigma(r^*, \mu^*, t^*)
\]

\[
= \mathbb{E}_\theta \left[ \sum_{i=1}^{N} r^*_i(\theta) \left( \int_{V} (v_i - \phi^{\Lambda^i}_i(\theta_i)) \mu^*_i(v|\theta) F(dv) - \phi^{\Lambda^i}_i(\theta_i) \right) \right].
\]

Proof. See Appendix A.1. ■

Lemma 3 implies that the mechanism \((r^*, \mu^*, t^*)\) is a solution to (1). Furthermore, Lemma 1 and Lemma 2 imply that any other solution of (1) must be equivalent to \((r^*, \mu^*, t^*)\) with probability 1, save for the tie breaking rules that do not affect efficiency.

Now consider any efficient mechanism. As noted above, it is without loss to assume that this mechanism is \((r^*, \mu^*, t^*)\). To see that \((r^*, \mu^*, t^*)\) is equivalent to a PRYCE CAP market structure, consider the market structure \((S, r^P, \mu^P, t^P)\) as follows: \(S_i := \mathbb{R}_+\) for all \(i\);

\[r^P_i(s) := 1\{i \in \mathcal{E}^P(s)\},\]

for all \(s \in S\), where \(\mathcal{E}^P(s)\) is the largest solution (in terms of the strong set order) of

\[
\max_{\mathcal{E} \subseteq \{1, \ldots, N\}} \left( \int_{V} \max_{i \in \mathcal{E}} (v_i - s_i)^+ F(dv) - \sum_{i \in \mathcal{E}} s_i \kappa_i \right),
\]

for all \(s \in S\);

\[\mu^P_i(v|s) = \begin{cases} \frac{1}{|\mathbb{M}(v, s)|}, & \text{if } v_i \geq s_i \text{ and } i \in \mathbb{M}(v, s) \\ 0, & \text{otherwise} \end{cases},\]

where \(\mathbb{M}(v, s) := \arg\max_{j \in \mathcal{E}^P(s)} \{v_j - s_j\}\); and

\[t^P_i(s) := s_i \mathbb{E}_{\theta_i} \left[ r^P_i(s_i, \phi^{-1}_i(\theta_{-i})) \int_{V} \mu^P_i(v|s_i, \phi^{-1}_i(\theta_{-i})) F(dv) \right] - \tau^*_i((\phi^A_i)^{-1}(s_i)),\]

where

\[\tau^*_i(\theta_i) := \phi^A_i(\theta_i) \mathbb{E}_{\theta_{-i}} \left[ r^*_i(\theta) \int_{V} \mu^*_i(v|\theta) F(dv) \right] - T^*_i(\theta_i),\]

and \((\phi^A_i)^{-1}(s_i) := \inf\{\theta_i \in \Theta_i|\phi^A_i(\theta_i) \geq s_i\}\), for all \(i\) and for all \(s_i \in S_i\).
Notice that for any $i$, any $s_{-i} \in S_{-i}$, and any $s_i, s_i' \in S_i$, if $s_i > s_i'$ and $i \in \mathcal{E}^P(s_i, s_{-i})$, it must be that $i \in \mathcal{E}^P(s_i', s_{-i})$. Thus, for any $i$ and for any $s_{-i} \in S_{-i}$, there exists $\bar{p}_i(s_{-i}) \in \mathbb{R}_+ \cup \{\infty\}$ such that $i \in \mathcal{E}^P(s_i, s_{-i})$ if and only if $s_i \leq \bar{p}_i(s_{-i})$. For this reason, $(S, r^P, \mu^P, t^P)$ is indeed a PRYCE CAP market structure.

The following lemma completes the proof.

**Lemma 4.** The PRYCE CAP market structure $(S, r^P, \mu^P, t^P)$ has a pure-strategy Bayes-Nash equilibrium $\sigma^P$ that induces the same outcome as $(r^*, \mu^*, t^*)$.

**Proof.** See Appendix A.2. ■

## 5 PRYCE CAP Example and Properties

### 5.1 PRYCE CAP Example

Suppose that the number of potentially active firms $N = 2$ and that consumer values and firm types $v_1, \theta_1, v_2, \theta_2 \in [0, 1]$ are independently drawn from a uniform distribution. Suppose further that the commonly known fixed cost parameters $\kappa_1 = \kappa_2 = 1$ and that the Pareto weight functions $\Lambda_1(x) = \Lambda_2(x) = x$ for all $x \in [0, 1]$. Then, the price cap functions $\bar{p}_1$ and $\bar{p}_2$ and the set $\mathcal{E}^P$ can be depicted by Figure 1.

Figure 1 illustrates the tight link between the yardstick price caps and the sets of firms optimally granted market entry. In the figure, the set of (undominated) prices $[0, 1]^2$ is partitioned into four regions, where each region of $(s_1, s_2)$ is mapped into different values of $\mathcal{E}^P(s_1, s_2)$. As a result, the boundaries of the regions define the yardstick price caps. The gold-orange curve represents firm 2’s price cap $\bar{p}_2$ as a function of $s_1$, and the blue-green curve represents firm 1’s price cap $\bar{p}_1$ as a function of $s_2$. Given firm 1’s published price $s_1$, firm 2 is excluded from the market if it posts a price $s_2 > \bar{p}_2(s_1)$. Similarly, given firm 2’s published price $s_2$, firm 1 is excluded from the market if it posts a price $s_1 > \bar{p}_1(s_2)$. Notice that both firms operate in the market if both publish relatively low prices, and both are restricted from entering if both publish relatively high prices. If one firm posts too high a price relative to the second, the first firm is excluded, whereas the second can enter.

Focusing on the behavior of the price caps in the figure, one can observe the two caps initially increasing in the other firm’s price. As the competing firm publishes a higher price,
the restriction on the other firm’s price loosens, consistent with the yardstick nature of the price ceiling. Once the competing firm’s price exceeds a certain value, though, the other firm’s price cap flattens, becoming independent of the competing firm’s choice. This change in pattern is from the competing firm no longer operating in the market precisely because its high price denied it entry. At that point, the price cap of the other firm remains fixed and its authorization for business depends only on its own published price.

5.2 Yardstick Price Cap Properties

The properties of the yardstick price cap just described are not special to the assumptions of two firms or uniformly distributed consumer values. Under the broader assumption that consumers’ values \( \{v_i\}_{i=1}^N \) are i.i.d., the next proposition explains that a firm’s price ceiling rises when competing firms submit higher prices. Moreover, a firm can guarantee itself entry if it submits a price below a certain threshold; and it can guarantee itself no entry if it submits a price above another threshold.

**Proposition 1.** Suppose that \( \{v_i\}_{i=1}^N \) are i.i.d. and that \( \kappa_i = \kappa \) for all \( i \). Consider any
efficient PRYCE CAP market structure and let $\bar{p}_i : S_{-i} \to \mathbb{R}_+ \cup \{\infty\}$ denote the yardstick price cap for firm $i$. Then,

1. For any price vector $s \in \mathbb{R}_+^N$, $\bar{p}_i(s_{-i}) \leq \bar{p}_j(s_{-j})$ if and only if $s_i \geq s_j$, for all $i, j \in \{1, \ldots, N\}$.

2. For any $i \in \{1, \ldots, N\}$ and for any $s_{-i} \in \mathbb{R}_+^{N-1}$, $\bar{p}_i(s_{-i}) \in [s, \bar{s}]$ for some $0 \leq s \leq \bar{s} < \infty$.

Proof. See Appendix B.1.

An immediate consequence of Proposition 1 is that the firm publishing the lowest price faces the highest price ceiling. This relation implies that the price a firm publishes has two effects on its eligibility to operate in an efficient PRYCE CAP market structure. The first is a direct effect: A lower submitted price is more likely to be below the firm’s price ceiling and grant the firm the right to sell. The second is a yardstick effect: A lower submitted price, other things equal, means the firm will face a higher price ceiling compared to its competitors, which can be more easily met.

6 Conclusion

In economies with incomplete information about firm costs and where consumers make discrete choices over goods, we show that among infinitely many market structures, the socially efficient ones are equivalent to price competition, but with lump-sum transfers and yardstick price caps. We refer to these market structures as PRYCE CAP market structures, and they can be implemented without knowledge of individual consumer preferences, realized firm costs, or firm conduct.

To implement a PRYCE CAP market structure, a planner, we presume, has power to verify and enforce competition exclusively on price, regardless of the kinds of complicated competitive conduct that might already prevail in the market. But upholding price competition is not the unique way to achieve efficiency. For certain markets, a clever selection of lump-sum transfers alone might convert an existing inefficient market structure into an efficient one. But administering such creative transfers would likely be intractable, and
the planner would need unearthly knowledge of the competitive game that firms engage. A significant contribution of PRYCE CAP market structures is that they require no such awareness, and they apply to a broad range of potential market environments.

Hence, if the planner can verify and enforce price competition, the search for efficiency ends with PRYCE CAP market structures. In practice, though, a planner might lack such powers entirely or wield them imperfectly. A natural implication of our result is that social efficiency is more easily achieved in markets where a planner can plausibly maintain price competition. Or rather, more realistically, markets where posting prices already drives the nature of competition are better candidates for reaching efficiency.
Appendix

A  Omitted Proofs for Section 4

A.1  Proof of Lemma 3

Proof. We first show that, for all $i$,

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}} \left[ r_i^*(\theta_i, \theta_{-i}) \left( \int_V \mu_i^*(v|\theta_i, \theta_{-i}) F(dv) + \kappa_i \right) \right]$$

is nonincreasing. To see this, notice that for any $i$ and for any $\theta \in \Theta$

$$\int_V \mu_i^*(v|\theta) F(dv) = \int_V 1\{\phi_i^A_i(\theta_i) \leq \phi_i^A_i(\theta_j) + v_i - v_j, \forall j \in \mathcal{E}^*(\theta), j \neq i\} F(dv).$$

Moreover, notice that for any $i$, for any $\theta_{-i} \in \Theta_{-i}$, and for any $\theta_i, \theta'_i \in \Theta_i$ with $\theta'_i < \theta_i$, $i \in \mathcal{E}^*(\theta_i, \theta_{-i})$ implies $i \in \mathcal{E}^*(\theta'_i, \theta_{-i})$. Together with the fact that $\phi_i^A_i$ is nondecreasing, it then follows that both (5) and $r_i^*$ are nonincreasing functions of $\theta_i$ for all $\theta_{-i} \in \Theta_{-i}$. Therefore, (4) is indeed nonincreasing.

Furthermore, by definition of $(r^*, \mu^*)$, for any $(r, \mu)$ such that the function

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}} \left[ r_i(\theta_i, \theta_{-i}) \left( \int_V \mu_i(v|\theta_i, \theta_{-i}) F(dv) + \kappa_i \right) \right]$$

is nonincreasing, it must be that

$$\mathbb{E}_\theta \left[ \sum_{i=1}^N r_i(\theta) \left( \int_V (v_i - \phi_i^A_i(\theta_i)) \mu_i(v|\theta) F(dv) - \phi_i^A_i(\theta_i) \kappa_i \right) \right]$$

$$\leq \mathbb{E}_\theta \left[ \sum_{i=1}^N r_i^*(\theta) \left( \int_V (v_i - \phi_i^A_i(\theta_i)) \mu_i^*(v|\theta) F(dv) - \phi_i^A_i(\theta_i) \kappa_i \right) \right].$$

Thus, $(r^*, \mu^*)$ is a solution to (2).

Lastly, by Lemma 2, since (4) is nonincreasing and is measurable with respect to $\phi_i^A_i$ for all $i$, we have

$$\int_{\Theta_i} \phi_i^A_i(\theta_i) \mathbb{E}_{\theta_{-i}} \left[ r_i^*(\theta) \left( \int_V \mu_i^*(v|\theta) F(dv) + \kappa_i \right) \right] G_i(d\theta_i)$$

$$= \int_{\Theta_i} \theta_i \mathbb{E}_{\theta_{-i}} \left[ r_i^*(\theta) \left( \int_V \mu_i^*(v|\theta) F(dv) + \kappa_i \right) \right] G_i(d\theta_i)$$

$$+ \int_{\Theta_i} (G_i(\theta_i) - \Lambda_i(\theta_i)) \mathbb{E}_{\theta_{-i}} \left[ r_i^*(\theta) \left( \int_V \mu_i^*(v|\theta) F(dv) + \kappa_i \right) \right] d\theta_i$$

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Therefore,
\[
\mathbb{E}_\theta \left[ \sum_{i=1}^{N} r_i^*(\theta) \left( \int_{V} (v_i - \phi_i^{\Lambda_i}(\theta_i)) \mu_i^*(v|\theta) F(dv) - \phi_i^{\Lambda_i}(\theta_i) \kappa_i \right) \right]
\]
\[
= \mathbb{E}_\theta \left[ \sum_{i=1}^{N} r_i^*(\theta) \int_{V} v_i \mu_i^*(v|\theta) F(dv) \right] - \sum_{i=1}^{N} \left\{ \int_{\Theta_i} \phi_i^{\Lambda_i}(\theta_i) \mathbb{E}_{\theta_{-i}} \left[ r_i^*(\theta) \left( \int_{V} \mu_i^*(v|\theta) F(dv) + \kappa_i \right) \right] G_i(d\theta_i) \right\}
\]
\[
= \mathbb{E}_\theta \left[ \sum_{i=1}^{N} r_i^*(\theta) \int_{V} v_i \mu_i^*(v|\theta) F(dv) \right] - \int_{\Theta_i} \left( G_i(\theta_i) - \Lambda_i(\theta_i) \right) \mathbb{E}_{\theta_{-i}} \left[ r_i^*(\theta) \left( \int_{V} \mu_i^*(v|\theta) F(dv) + \kappa_i \right) \right] d\theta_i
\]
\[
= \sum(r^*, \mu^*, t^*) + \sum_{i=1}^{N} \int_{\Theta_i} \Pi(\theta_i| r^*, \mu^*, t^*) \Lambda_i(d\theta_i),
\]
as desired. ■

A.2 Proof of Lemma 4

Proof. Consider the mechanism \((S, r^P, \mu_i^P, t^P)\). First, notice that by definition of \(r^P\) and \(\mu_i^P\), for all \(\theta \in \Theta\) and for all \(i\),

\[
r_i^P(\phi_i^{\Lambda_i}(\theta_1), \ldots, \phi_i^{\Lambda_N}(\theta_N)) = r_i^*(\theta_1, \ldots, \theta_N);
\]

and

\[
\mu_i^P(v|\phi_i^{\Lambda_i}(\theta_1), \ldots, \phi_i^{\Lambda_N}(\theta_N)) = \mu_i^*(v|\theta_1, \ldots, \theta_N),
\]

for all \(v \in V\). Moreover, by Lemma 1, for each \(i\) and for any interval \([\theta_i^1, \theta_i^2]\) on which \(\phi_i^{\Lambda_i}\) is constant, \(T_i^*\) is also constant. Therefore, for any \(i\) and for any \(\theta_i \in \Theta_i\), if \(\theta_i\) belongs to an interval \([\theta_i^1, \theta_i^2]\) on which \(\phi_i^{\Lambda_i}\) is a constant, then \((\phi_i^{\Lambda_i})^{-1}(\phi_i^{\Lambda_i}(\theta_i)) = \theta_i^2 = \theta_i\). Thus, for any \(i\) and for any \(\theta \in \Theta\),

\[
t_i^P(\phi_i^{\Lambda_i}(\theta_1), \ldots, \phi_i^{\Lambda_N}(\theta_N)) = \mathbb{E}_{\theta_{-i}} \left[ \phi_i^{\Lambda_i}(\theta_i) r_i^P(\phi_i^{\Lambda_i}(\theta_i), \phi_i^{\Lambda_{-i}}(\theta_{-i})) \int_{V} \mu_i^P(v|\phi_i^{\Lambda_i}(\theta_i), \phi_i^{\Lambda_{-i}}(\theta_{-i})) F(dv) \right] - \tau_i^*(\theta_i)
\]
\[
= \mathbb{E}_{\theta_{-i}} \left[ \phi_i^{\Lambda_i}(\theta_i) r_i^*(\theta) \int_{V} \mu_i^*(v|\theta) F(dv) \right] - \tau_i^*(\theta_i)
\]
\[
= T_i^*(\theta_i)
\]
\[
= t_i^*(\theta_1, \ldots, \theta_N),
\]

where \(\phi_i^{\Lambda_{-i}} := (\phi_i^{\Lambda_1}, \ldots, \phi_i^{\Lambda_{i-1}}, \phi_i^{\Lambda_{i+1}}, \ldots, \phi_i^{\Lambda_N})\).

It then remains to show that the strategy profile where each firm \(i\) with type \(\theta_i\) chooses \(\phi_i^{\Lambda_i}(\theta_i)\) is a Bayes-Nash equilibrium in the game induced by \((S, r^P, \mu_i^P, t^P)\). Indeed, for any firm \(i\), any type \(\theta_i \in \Theta_i\), and for any \(s_i \in \phi_i^{\Lambda_i}(\Theta_i)\), given that all other firms follow the strategy \(\phi_i^{\Lambda_{-i}} = (\phi_i^{\Lambda_1}, \ldots, \phi_i^{\Lambda_{i-1}}, \phi_i^{\Lambda_{i+1}}, \ldots, \phi_i^{\Lambda_N})\), let \(\theta_i' \in \Theta_i\) be such that \(\phi_i^{\Lambda_i}(\theta_i') = s_i\). We
then have
\[
\begin{align*}
\mathbb{E}_{q_{-i}} & \left[ t_i^P(\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^P(\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i \left( \int_{\mathcal{V}} \mu_i^P(v|\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})))F(dv) + \kappa_i \right) \right] \\
& = \mathbb{E}_{q_{-i}} \left[ t_i^P(\theta_i, \theta_{-i}) - r_i^P(\theta_i, \theta_{-i})\theta_i \left( \int_{\mathcal{V}} \mu_i^P(v|\theta_i, \theta_{-i}))F(dv) + \kappa_i \right) \right] \\
& \geq \mathbb{E}_{q_{-i}} \left[ t_i^P(\theta'_i, \theta_{-i}) - r_i^P(\theta'_i, \theta_{-i})\theta_i \left( \int_{\mathcal{V}} \mu_i^P(v|\theta'_i, \theta_{-i}))F(dv) + \kappa_i \right) \right] \\
& = \mathbb{E}_{q_{-i}} \left[ t_i^P(s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^P(s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i \left( \int_{\mathcal{V}} \mu_i^P(v|s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})))F(dv) + \kappa_i \right) \right],
\end{align*}
\]
where the inequality follows from the fact that \((r^*, \mu^*, t^*)\) is incentive compatible. Meanwhile, it is easy to verify that for any firm \(i\), any type \(\theta_i \in \Theta_i\), and for any \(s_i \notin \phi_i^{\Lambda_i}(\Theta_i)\), given that all other firms follow the strategy \(\phi_{-i}^{\Lambda_{-i}}\),
\[
\begin{align*}
\mathbb{E}_{q_{-i}} & \left[ t_i^P(\phi_i^{\Lambda_i}(\theta'_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^P(\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i \left( \int_{\mathcal{V}} \mu_i^P(v|\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})))F(dv) + \kappa_i \right) \right] \\
& \geq \mathbb{E}_{q_{-i}} \left[ t_i^P(\phi_i^{\Lambda_i}(\theta'_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^P(s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i \left( \int_{\mathcal{V}} \mu_i^P(v|s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})))F(dv) + \kappa_i \right) \right].
\end{align*}
\]
Together, it then follows that \((\phi_i^{\Lambda_i}, \ldots, \phi_{N}^{\Lambda_{-i}})\) is indeed a Bayes-Nash equilibrium in the game induced by \((S, r^P, \mu^P_i, t^P)\). This completes the proof. \(\blacksquare\)

## B Omitted Proof for Section 5

### B.1 Proof of Proposition 1

**Proof.** From the proof of Theorem 1, for each \(i \in \{1, \ldots, N\}\) and for any \(s \in \mathbb{R}^N_+\), firm \(i \in \mathcal{E}^P(s_i, s_{-i})\) if and only if \(s_i \leq \bar{p}_i(s_{-i})\), where \(\mathcal{E}^P\) is the (largest) solution of
\[
\max_{\varepsilon \subseteq \{1, \ldots, N\}} \left( \int_0^\infty \cdots \int_0^\infty \max_{v_i \in \varepsilon}(v_i - s_i)F(dv_1) \cdots F(dv_N) - \sum_{i \in \varepsilon} s_i \kappa_i \right).
\]
As a result, there must exists \(\bar{p} : \mathbb{R}^{N-1}_+ \rightarrow \mathbb{R}_+ \cup \{\infty\}\) such that \(\bar{p}_i(s_{-i}) = \bar{p}(s_{-i})\) for all \(i\) and for all \(s \in \mathbb{R}^N_+\). We claim that \(\bar{p}\) is nondecreasing in each argument. Indeed, for any \(i\) and for any \(s, s' \in \mathbb{R}^N_+\), such that \(s_i = s'_i\) and \(s_j \leq s'_j\) for some \(j \neq i\), if \(i \in \mathcal{E}^P(s)\), then it must be that \(i \in \mathcal{E}^P(s')\) as well. Therefore, it must be that \(\bar{p}(s_{-i}) \leq \bar{p}(s'_{-i})\), as desired. Since \(\bar{p}\) is nondecreasing in every component, for any \(i, j \in \{1, \ldots, N\}\) with \(i \neq j\), and for any \(s \in \mathbb{R}^N_+\) with \(s_i \geq s_j\), it must be that \(\bar{p}(s_{-j}) \geq \bar{p}(s_{-i})\), as desired.

Meanwhile, notice that for any \(i \in \{1, \ldots, N\}\) and for any \(s_{-i} \in \mathbb{R}^{N-1}_+\), if \(s_i = 0\), then it
must be that \( i \in \mathcal{E}(s_i, s_{-i}) \). In contrast, since for any \( \mathcal{E} \subseteq \{1, \ldots, N\} \) such that \( i \notin \mathcal{E} \),

\[
\lim_{s_i \to \infty} \sup_{s_{-i} \in \mathbb{R}^N_+} \left[ \int_0^\infty \cdots \int_0^\infty \left[ \max_{j \in \mathcal{E} \cup \{i\}} (v_j - s_j)^+ - \max_{j \in \mathcal{E}} (v_j - s_j)^+ \right] F(dv_1) \cdots F(dv_N) - s_i \kappa \right] < 0,
\]

there must exist \( \bar{s} \) such that \( i \notin \mathcal{E}(s) \) whenever \( s_i \leq \bar{s} \), for all \( s \in \mathbb{R}^N_+ \). This completes the proof. ■

References


Online Appendix

C Omitted Details for Section 2.3

Details for the Quantity Competition Market Structure: Let the strategy space be $S = [0, 1]^N$, and let entry probabilities be $r_i(s) = 1$ for all $i$ and for all $s \in S$. Furthermore, since $F$ is atomless, the function $(p_1, \ldots, p_N) \mapsto \int_V 1\{v_i \geq p_i, \forall i\} F(\mathrm{d}v)$ is continuous and nondecreasing and has value 1 at $p_1 = \ldots = p_N = 0$ and value 0 at $p_1 = \ldots = p_N = \max_i \{\max(\text{proj}_i V)\}$. Therefore, for any $s \in S$ such that $\sum_j s_j \leq 1$, there exists $\{p_i(s)\}_{i=1}^N$ such that

$$\int_V 1\{v_i \geq p_i(s), \forall i\} F\mathrm{d}v = \sum_{j=1}^N s_j.$$ 

Take any of such functions, and, for each $i$, extend $p_i$ to be defined on the entire $S$ by letting $p_i(s) = 0$ for all $s \in S$ such that $\sum_j s_j > 1$.

Now let

$$\mu_i(v|s) := \begin{cases} \frac{s_i}{\sum_j s_j}, & \text{if } v_j \geq p_i(s), \forall j \\ 0, & \text{otherwise} \end{cases},$$

for all $i$, for all $v \in V$, and for all $s \in S$, and let the revenues be defined as $t_i(s) = \int_V \mu_i(v|s) F(\mathrm{d}v)$.

Details for the Entry Deterrence Market Structure: Let $S_1 := \mathbb{R}_+$ and let $S_i := \{e_i : S_1 \to \{0, 1\}\} \times \{p_i : S_1 \to \mathbb{R}_+\}$, so that a strategy $s_i$ for firm $i > 1$ can be written as $s_i = (e_i, p_i)$. Define the revenues as $t_i(s_1, (e_i, p_i)) = p_i(s_1) \int_V \mu_i(v|s) F(\mathrm{d}v)$, and let the entry probabilities be $r_1(s_1, (e_i, p_i)) = 1$ and $r_i(s_1, (e_i, p_i)) = e_i(s_1)$, for all $s \in S$ and for all $i > 1$. Finally, let $\mu$ be defined as

$$\mu_i(v|s_1, (e_i, p_i)) = \begin{cases} \frac{1}{|\mathcal{M}(v, s_1, (e_i, p_i))|}, & \text{if } v_i - p_i(s_1) = \max_j \{(v_j - p_j(s_1))^+\} \text{ and } r_i(s_1, (e_i, p_i)) = 1 \\ 0, & \text{otherwise} \end{cases},$$

for all $i \in \{1, \ldots, N\}$ and $(s_i, (e_i, p_i)) \in S$, where $\mathcal{M}(v, s_1, (e_i, p_i)) := \arg\max_i \{(v_i - p_i(s_1))\}$ and $p_i(s_1) = s_1$. 

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