Market Structure Design*

Kai Hao Yang
Cowles Foundation, Yale University
kaihao.yang@yale.edu

Alexander K. Zentefis
Yale School of Management
alexander.zentefis@yale.edu

March 24, 2021

Abstract

The structure of a market describes the nature of competition between participating firms. This paper introduces a framework for designing market structures and characterizes the socially efficient ones. We define a market structure as the set of firms' strategies and mappings from those strategies to market entry rules, an allocation of products to consumers, and firm revenues. We show that efficient market structures are equivalent to price competition with transfers and yardstick price caps that depend on the published prices of competing firms. As a result, efficient market structures can be implemented without prior knowledge of individual consumer preferences, firms' realized costs, or firm conduct.

JEL classification: D40, D82, L10, L51

Keywords: market structure, industrial organization, mechanism design, regulation

*We are grateful to Florian Ederer, Gary Gorton, Yunzhi Hu, Fiona Scott Morton, and Jidong Zhou for helpful feedback and comments.

1 Introduction

In neoclassical economics, the *structure* of a market describes the nature of competition between participating firms (McNulty, 1968; Baumol, 1982). Core elements of a market structure include the ease of entry and exit, the number of active sellers, the distribution of market shares, and firm conduct (i.e., sellers' available competitive strategies and how those strategies translate into market equilibrium outcomes).

Upon first thought, one might conclude that a market structure requires no "design" by an outsider. To achieve a social welfare optimum, perhaps *laissez-faire* is the correct principle. One notable contrary example that demonstrates the advantage of intervention is that of a monopolist who operates with private information about cost. David Baron and Roger Myerson studied the optimal regulation of such a firm and justified the use of an interventionist policy to maximize expected social welfare (Baron and Myerson, 1982). A second example is Steven Salop's circular model of monopolistic competition (Salop, 1979). The decentralized equilibrium with costly entry features an excessive number of product varieties compared to the optimum. That loss in welfare can be resolved by a regulator increasing the cost of firm creation.

Having argued for the need for market structure design, it is next important to prescribe some properties that bolsters its practical use. It would be helpful that the designer not require omniscience of the market to implement a market structure. Individual consumer preferences, the realized costs of production, and the strategic game that firms partake in may excusably escape a designer's competence. The first two could be estimated imperfectly, but understanding the complete nature of the existing firm conduct is especially challenging. Bounded knowledge should not constrain the achievement of an optimal design.

A second useful property is that the design and the regulatory policy to implement the design are simple. When outside interference is warranted to improve welfare, the design and the policy should be readily understood by all parties involved (both the regulator and the regulated), who presumably have a functioning grasp of economics. Part of being simple is also being familiar. The design and the implementation policy is better off avoiding components that are completely alien to regulatory history.

This paper provides a framework for designing market structures, characterizes the socially efficient ones, and shows that these efficient structures can be designed in a simple way that requires no omniscience of the market or its players. In our setting, a fixed number of firms selling heterogeneous products have the potential to operate. A firm's production costs are private information, drawn from a general distribution, and are independent from every other firm's. A continuum of consumers have unit demand and heterogeneous tastes over products. With these cost and preference distributions, we formally define a *market structure* as a mapping that assigns firms' strategies to a tuple of (i) probabilities that each firm enters the market, (ii) an allocation of products to consumers among firms who enter, and (iii) firm revenues.

Given a market structure, the timing of the model begins with firms privately observing their own costs, choosing strategies, and finally receiving an ex-post payoff in the form of profits. In effect, a market structure defines a Bayesian game in which each firm has a private type, a strategy space, and a profit payoff function. To be more concrete, consider a setting akin to Bertrand (1883) with a homogenous product. Here, the market structure entails price competition: Firms post prices simultaneously, and consumers buy from firms with the lowest price whenever there are gains from trade.

Having established what a market structure is, we then describe the socially efficient market structures. When measuring efficiency, we focus on the interim period in which each firm personally knows its realized production costs, but holds only expectations of its competitors' costs. That way, we can be assured that each firm earns no less under an efficient market structure than under any other market structures without intervention.

In our first theorem, we show that among infinitely many possible market structures, the efficient ones are equivalent to the following class: price competition with transfers and yardstick price caps. We label this class PRYCE CAP market structures. In this class, firms strategize and keep control over price. But each firm faces an individual price cap that depends on the posted prices of the other firms. We use the word "yardstick" in a way similar to Shleifer (1985)'s use of the word, in that a firm's regulation depends on characteristics of other firms. Embedded in a firm's price cap is an operating license (i.e., a right to enter the market). A firm is granted a license, if it proposes to set a price under its price cap.[1,2] Among firms that are granted licenses, a consumer selects a firm (i.e., transacts with it), if the consumer prefers the firm's product at the posted price above all other competitors.

Every efficient market structure can be implemented by a PRYCE CAP market structure, which utilizes only transfers and price caps. Hence, designing an efficient market structure is familiar to regulatory experience—making it simple. Designing it also requires no omniscient knowledge of individual consumer preferences, realized production costs, or firm conduct. The designer need only observe the public prices that firms announce.

Looking across all possible market structures, we unveil the price cap's central role in market structure design. Doing so establishes its primacy in efficient regulatory regimes and rationalizes its use in practice. The literature has thus far only endogenized the price cap as a regulatory tool in restricted settings, such as that of monopoly (e.g., Lewis and Sappington, 1989; Schmalensee, 1989).

In a corollary to our theorem, we show that for any existing market structure, either it already is interim Pareto efficient, or there exists a PRYCE CAP market structure that Pareto dominates it. The corollary thus provides a straightforward, precise recipe to make any market structure interim efficient if it is not already: price competition plus transfers and yardstick price caps. All firms would be open to participating in the new market structure, as all would earn at least as much as they had under the existing one.

Having demonstrated the efficiency of PRYCE CAP market structure, we then ask: when is it possible for market structures to be socially efficient on their own, without outside interference? To answer this question, we introduce a *laissez-faire* market structure. In this structure, a strategy profile among firms leads to a vector of prices for their products. The revenue of a firm, conditional on it competing in the market, is its product's price times

[1]Our yardstick price cap has commonality with the "coopetition" price cap of Rey and Tirole (2019). While the yardstick price cap is not voluntarily set by the firms themselves as in Rey and Tirole (2019), it similarly is determined by firms' joint strategic decisions.

[2]The license to operate embedded in the price cap differs from firms competing in an auction for the right to supply a product (as in Demsetz, 1968; Laffont and Tirole, 1987; Riordan and Sappington, 1987; Kjerstad and Vagstad, 2000) or to enter a research tournament (as in Taylor, 1995; Fullerton and McAfee, 1999; Che and Gale, 2003). Here, firms are not competing over a single object. Potentially all firms may be granted a license and enter the market. The operating license is essentially a multi-good auction (e.g., Armstrong, 2000) where the bidders have unit demand.

quantity supplied. Transfers of any other form are not allowed. Consumers, for their part, are only required to be sequentially rational. That is, if a consumer buys from a firm, the person must value the product at least as much as the price she pays for it. Understanding when this kind of "free market" structure is efficient is crucial to identifying when regulating a market is necessary, if at all.

In our second theorem, we prove that a *laissez-faire* market structure is efficient only if firms have no private information. Few strategic industrial settings in theory and arguably none in reality satisfy this requirement. For example, while Bertrand price competition with complete information achieves allocative efficiency, any perturbation to firms' information structure immediately renders that market structure inefficient. This result, then, suggests that regulation, if done correctly, is generically necessary for all industrial markets. And the paper identifies the right tools on how to do it.

Our framework applies to a broad range of industrial settings, as it can accommodate a variety of strategies that firms are thought to employ when competing. This expansive treatment is important, as the nature of competition can differ substantially between industries. As Berry et al. (2019) note, "prices are determined in the food distribution industry via second price auction, in health care via bilateral bargaining, and in retail as posted prices." Our model for market structures nests each of these examples and more. Among others are pure monopoly, competing on price á la Bertrand (1883), competing on quantity á la Cournot (1838), competition over differentiated products (Perloff and Salop, 1985), and consumer search (Varian, 1980; Narasimhan, 1988; Armstrong and Vickers, 2019).

The framework builds on existing studies of market regulation with asymmetric information. Literature in this area thus far has more or less fixed the market structure in the analysis rather than treat it as an object to design from scratch. For example, a great deal of work has centered on managing the behavior of a monopolist who holds private knowledge about costs or demand (e.g., Baron and Myerson, 1982; Guesnerie and Laffont, 1984; Laffont and Tirole, 1986; Lewis and Sappington, 1988; Armstrong and Sappington, 2004).

Even when researchers account for multiple firms, they have preselected market structures to compare welfare, such as monopoly against duopoly (e.g., Auriol and Laffont, 1992; Armstrong and Sappington, 2006). Or, researchers have searched for the optimal number of suppliers of a single product (see Mankiw and Whinston, 1986; Vickers, 1995 and work on spectrum auctions such as McMillan, 1994; Milgrom, 1998; Hoppe et al., 2006). Lastly, they have explored whether a single firm should supply multiple products or rather, different firms supply different products (e.g., Palfrey, 1983; Baron and Besanko, 1992; Dana Jr, 1993; Gilbert and Riordan, 1995; Severinov, 2003).

In contrast, we start with as bare an architecture as we can imagine. We assume general distributions for consumer tastes and firm costs. The number of active firms that enter is endogenous. Regarding the market structure, we only take as given the existence of a functioning *market*—an institution "for the consummation of transactions," as George Stigler put it (Stigler, 1957). From here, we search among an extensive set of possibilities for the desirable game of competition among industry players, so as to maximize welfare.

Our approach is closer to the mechanism design literature that considers all possible interacting channels through which market equilibrium outcomes can be determined. In particular, our model can be thought of as a generalization of Baron and Myerson (1982) that considers an economy with more than one firm. With multiple firms, the allocation space becomes much richer compared with Baron and Myerson (1982). In our setting, the relevant allocation is the entire distribution of matches of consumers and products, instead of a one-dimensional demand quantity.[3]

The paper proceeds as follows. Section 2 describes the model. Section 3 states the main result: Every efficient market structure is equivalent to a PRYCE CAP market structure. Section 4 proves the main result. Section 5 gives an illustrative example and describes *laissez-faire* market structures. Section 6 concludes.

2 Model

2.1 Primitives

A number $N \geq 1$ of firms produce N heterogeneous products. Each firm i has cost function $C_i(q) = \theta_i(q + \kappa_i)$, where q is quantity and $\kappa_i \geq 0$ is commonly known. Meanwhile, $\theta_i \geq 0$ is the firm's private information. The values $\theta = (\theta_i)_{i=1}^N \in \mathbb{R}^N$ are independent and θ_i follows a

[3] See Amador and Bagwell (2021) and Guo and Shmaya (2019) for other recent papers related to Baron and Myerson (1982).

distribution G_i. Assume that $\text{supp}(G_i) \subset \mathbb{R}_+$ is compact and denote $\overline{\theta}_i := \max \text{supp}(G_i)$ and $\underline{\theta}_i := \min \text{supp}(G_i)$ so that $\text{co}(\text{supp}(G_i)) = \Theta_i := [\underline{\theta}_i, \overline{\theta}_i]$. A unit mass of consumers stand ready to purchase. Each consumer has unit demand and heterogeneous values $\mathbf{v} \in V \subseteq \mathbb{R}_+^N$, so that a consumer with value vector $\mathbf{v} = (v_1, \ldots, v_N)$ has value v_i for firm i's product, where V is a hypercube in $\mathbb{R}_+^N$. The consumers' values are distributed according to measure $F \in \Delta(V)$.

2.2 Market Structure

A *market structure* is a tuple $(S, r, \boldsymbol{\mu}, t)$ that assigns firms' strategies to (i) market entry probabilities, (ii) an allocation of products to consumers, and (iii) firm revenues. Formally, $S = \prod_{i=1}^N S_i$, where S_i is a measurable space that describes firm i's strategies. The measurable function $r : S \to [0,1]^N$ maps firms' strategy profiles to firms' probabilities of entering the market. Similarly, $t : S \to \mathbb{R}^N$ maps firms' strategy profiles to firms' revenues. Finally, let A be the collection of measurable functions $\mathbf{v} \mapsto (\mu_i(\mathbf{v}))_{i=1}^N$ such that $\sum_i \mu_i(\mathbf{v}) \leq 1$. The product allocation rule $\boldsymbol{\mu}$ is a measurable function from S to A, mapping firms' strategy profiles to an allocation of products among consumers .

The interpretation of a market structure is that, given a strategy profile $s \in S$, $r_i(s) \in [0,1]$ is the probability that firm i enters the market; $\boldsymbol{\mu}_i(\mathbf{v}|s) \in [0,1]$ is the share of consumers with value vector $\mathbf{v}$ who buy the product from firm i, conditional on firm i being in the market; and $t_i(s)$ is the revenue of firm i. We normalize the firms' outside options to zero, and require that any market structure must allow an opt-out option $s_0 \in S_i$ such that $t_i(s_0, s_{-i}) = \boldsymbol{\mu}_i(\mathbf{v}|s_0, s_{-i}) = r_i(s_0, s_{-i}) = 0$ for all i, for all $s_{-i} \in S_{-i}$, and for all $\mathbf{v} \in V$.

Given a market structure $(S, r, \boldsymbol{\mu}, t)$, the timing of events is as follows.

1. Types $\{\theta_i\}_{i=1}^N$ are drawn independently from $\{G_i\}_{i=1}^N$, and each firm privately observes its own cost.

2. Firms simultaneously choose s_i from S_i.

3. Each firm i receives (ex-post) payoff

$$\pi_i(s, \theta_i | S, r, \boldsymbol{\mu}, t) := t_i(s) - r_i(s)\theta_i \left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v}) + \kappa_i \right).$$

Notice that a market structure $(S, r, \boldsymbol{\mu}, t)$ defines a Bayesian game where each firm i has private type $\theta_i \in \Theta_i$, strategy space S_i, and payoff function $\pi_i(s, \theta_i | S, r, \boldsymbol{\mu}, \boldsymbol{p})$.

2.3 Range of Competitive Games

Here we provide some examples of market structures. We also discuss the range of the competitive games that our framework accommodates.

Example 1 (**Monopoly**). The following market structure describes a monopoly market where firm $i = 1$ is a monopolist. The strategy spaces are $S = \mathbb{R}_+^N$; firm revenues are given by $t_i(s) := s_i \int_V \mu_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v})$, entry probabilities are $r_1(s_1) = 1$ and $r_j(s_j) = 0$ for all $j \neq i$, and the allocation of products to consumers is

$$\boldsymbol{\mu}_i(\mathbf{v}|s) = \begin{cases} 1, & \text{if } i = 1 \text{ and } v_i \geq s_i \\ 0, & \text{otherwise} \end{cases},$$

for all i and for all $s \in S$.

Under this market structure, only firm 1 operates in the market and chooses a monopoly price $s_1 \geq 0$ to sell to the consumers. Given price s_i, all the consumers with value $v_1 \geq s_1$ buys from firm 1 whereas all other consumers do not buy.

Example 2 (**Price Competition**). The following market structure describes a price competition model. Let the strategy spaces be $S := \mathbb{R}_+^N$, revenues be $t_i(s) = s_i \int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v})$, and entry probabilities be $r_i(s) = 1$ for all i and for all $s \in S$. Furthermore, define $\boldsymbol{\mu}$ as

$$\boldsymbol{\mu}_i(v|s) = \begin{cases} \frac{1}{|\mathbb{M}(\mathbf{v},s)|}, & \text{if } v_i - s_i = \max_j\{v_j - s_j\} \text{ and } v_i \geq s_i \\ 0, & \text{otherwise} \end{cases},$$

for all $i \in \{1, \ldots, N\}$ and for all $s \in S$, where $\mathbb{M}(\mathbf{v}, s) := \mathrm{argmax}_i\{(v_i - s_i)\}$.

Under this market structure, all N firms operate in the market and compete on the price margin. That is, each firm i sets price $s_i \geq 0$. After seeing firms' prices $s = (s_1, \ldots, s_N)$, consumers buy from the firm that gives them the highest surplus.

Notice that with different specifications of the value distribution F, this market structure corresponds to various canonical competition models. In particular, by assuming that $\mathbf{v}$ is perfectly correlated (i.e., $v_1 = \ldots, v_N = v$ with F-probability 1), we have the classical Bertrand competition (Bertrand, 1883) model (with private marginal costs); by assuming that $\mathbf{v}$ is independent, we have the model á la Perloff and Salop (Perloff and Salop, 1985); by assuming that $N = 2$ and that $\mathbf{v}$ is perfectly negatively correlated (i.e., $v_1 + v_2 = 1$ with F-probability 1), we have the Hotelling location model (Hotelling, 1929).

Example 3 (**Quantity Competition**). Suppose that F is atomless. The following market structure describes a quantity competition model. Let the strategy space be $S = [0,1]^N$, and let entry probabilities be $r_i(s) = 1$ for all i and for all $s \in S$. Furthermore, since F is atomless, the function

$$(p_1, \ldots, p_N) \mapsto \int_V \mathbf{1}\{v_i \geq p_i,\, \forall i\} F(\mathrm{d}\mathbf{v})$$

is continuous and nondecreasing, and has value 1 at $p_1 = \ldots = p_N = 0$ and value 0 at $p_1 = \ldots = p_N = \max_i\{\max(\mathrm{proj}_i V)\}$. Therefore, for any $s \in S$ such that $\sum_j s_j \leq 1$, there exists $\{\boldsymbol{p}_i(s)\}_{i=1}^N$ such that

$$\int_V \mathbf{1}\{v_i \geq \boldsymbol{p}_i(s),\, \forall i\} F(\mathrm{d}\mathbf{v}) = \sum_{j=1}^N s_j.$$

Take any such functions, for each i, extend $\boldsymbol{p}_i$ to be defined on the entire S by letting $\boldsymbol{p}_i(s) = 0$ for all $s \in S$ such that $\sum_j s_j > 1$.

Now let

$$\boldsymbol{\mu}_i(\mathbf{v}|s) := \begin{cases} \frac{s_i}{\sum_j s_j}, & \text{if } v_j \geq \boldsymbol{p}_i(s),\, \forall j \\ 0, & \text{otherwise} \end{cases},$$

for all i, for all $\mathbf{v} \in V$ and for all $s \in S$ and let the revenues be defined as $t_i(s) = \boldsymbol{p}_i(s) \int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v})$.

Under this market structure, each firm i chooses quantity s_i they wish to sell. Market prices (and hence allocation of the products) are determined through the *inverse demand* functions $\{\boldsymbol{p}_i\}_{i=1}^N$. Therefore, given any profile of chosen quantities $s = (s_1, \ldots, s_N)$, firm i sells

$$\int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v}) = \frac{s_i}{\sum_j s_j} \int_V \mathbf{1}\{v_j \geq \boldsymbol{p}_j(s),\, \forall j\} F(\mathrm{d}\mathbf{v}) = s_i$$

units at price $\boldsymbol{p}_i(s)$ if $\sum_j s_j \leq 1$, and sells $\frac{s_i}{\sum_j s_j}$ units at price 0 if $\sum_j s_j > 1$. This is strategically equivalent to a quantity competition game with inverse demand functions $\{\boldsymbol{p}_i\}_{i=1}^N$.[4]

Example 4 (**Entry Deterrence**). The following market structure describes a model where an incumbent (firm 1) can use price to deter entry of potential entrants á la Von Stackelberg (1934) sequential competition. Let firm 1's strategy space be $S_1 = \mathbb{R}_+$ and let all other firms' strategy space be $S_i = \{e_i : S_1 \to \{0,1\}\} \times \{\boldsymbol{p}_i : S_1 \to \mathbb{R}_+\}$ so that a strategy s_i for firm $i > 1$ can be written as $s_i = (e_i, \boldsymbol{p}_i)$. Let the revenues be $t_i(s_1, (e_i, \boldsymbol{p}_i)_{i=2}^N) = \boldsymbol{p}_i(s_1) \int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v})$; and let the entry probabilities be $r_1(s_1, (e_i, \boldsymbol{p}_i)_{i=2}^N) = 1$ and $r_i(s_1, (e_i, \boldsymbol{p}_i)_{i=2}^N) = e_i(s_1)$, for all $s \in S$ and for all $i > 1$. Finally, let $\boldsymbol{\mu}$ be defined as

$$\boldsymbol{\mu}_i(\mathbf{v}|s_1, (e_i, \boldsymbol{p}_i)_{i=2}^N) = \\ \begin{cases} \frac{1}{|\mathbb{M}(\mathbf{v}, s_1, (e_i, \boldsymbol{p}_i)_{i=2}^N)|}, & \text{if } v_i - \boldsymbol{p}_i(s_1) = \max_j\{(v_j - \boldsymbol{p}_j(s_1))^+\} \text{ and } r_i(s_1, (e_i, \boldsymbol{p}_i)_{i=1}^N) = 1 \\ 0, & \text{otherwise} \end{cases},$$

for all $i \in \{1, \ldots, N\}$ and $(s_i, (e_i, \boldsymbol{p}_i)_{i=2}^N) \in S$, where $\mathbb{M}(\mathbf{v}, s_1, (e_i, \boldsymbol{p}_i)_{i=2}^N) := \operatorname{argmax}_i\{(v_i - \boldsymbol{p}_i(s_1))\}$ and $\boldsymbol{p}_1(s_1) = s_1$.

Under this market structure. Firm 1 is an incumbent and chooses its price s_1 first. Afterward, all other firms observe firm 1's price s_1, and then they choose whether to enter the market, as well as what price to charge. That is, each firm $i > 1$ chooses an entry rule e_i and a pricing rule $\boldsymbol{p}_i$ as functions of firm 1's price. After entry decisions are made and prices are set, consumers then buy from the active firm that gives them the highest surplus.

Example 5 (**Promotional Sales and Consumer Search**). The following market structure describes a model with "captive consumers" and "shoppers". Let the action spaces be $S = \mathbb{R}_+^N$, revenues be $t_i(s) = s_i \int_V \mu_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v})$, and entry probabilities be $r_i(s) = 1$ for all

[4] The inverse demand functions $\{\boldsymbol{p}_i\}_{i=1}^N$ are defined implicitly through the intermediate value theorem, and there could be multiple sets of inverse demands that satisfy this condition. Such multiplicity embeds different complementarity structures. For instance, we may always define $\boldsymbol{p}_i$ by

$$\int_V \mathbf{1}\left\{v_i \geq \boldsymbol{p}_i\left(\sum_j s_j\right), \forall i\right\} F(\mathrm{d}\mathbf{v}) = \sum_j s_j$$

so that $\boldsymbol{p}_1 = \ldots = \boldsymbol{p}_N$. This would corresponds to the classical Cournot model (Cournot, 1838) (with private marginal cost).

$s \in S$ and for all i. Furthermore, let

$$\boldsymbol{\mu}_i(\mathbf{v}|s) = \begin{cases} \gamma_i + \left(1 - \sum_{j=1}^N \gamma_j\right) \frac{\mathbf{1}\{s_i \in \mathbb{M}(\mathbf{v},s)\}}{|\mathbb{M}(\mathbf{v},s)|}, & \text{if } v_i \geq s_i \\ 0, & \text{if } v_i < s_i \end{cases},$$

for all i, for all $\mathbf{v}$, and for all s, where $\mathbb{M}(\mathbf{v}, s) := \mathrm{argmax}_i\{v_i - s_i\}$, $\gamma_i \in [0, 1]$ for all i, and $\sum_{i=1}^N \gamma_i \leq 1$.

Under this market structure, each firm i has γ_i share of "captive" consumers that are loyal to firm i. These consumers only see firm i's price. Meanwhile, the remaining $1 - \sum_j \gamma_j$ share of consumers are "shoppers" who visit all the firms and thus observe all firms' prices. Notice that if $\mathbf{v}$ is perfectly correlated so that with F-probability 1, $v_1 = v_2 = \cdots = v_N$, this market structure describes the promotional sales model of Armstrong and Vickers (2019), which in turn nests the model of Varian (1980) and Narasimhan (1988).

Example 6 (**Reverse Auction with Many Buyers**). The following market structure describes a type of reverse auction where firms submit bids (prices) and firms with the lowest bid win and sell their products to all consumers with values above that bid. Pharmacy Benefit Managers (PBMs) are known to employ this kind of auction. The PBM holds an auction across drugs that treat a particular medical condition. The drugs of the winning manufacturers (the sellers) with the lowest bids are put on the PBM's formulary for doctors to prescribe and patients (the consumers) to purchase.[5] $S = \mathbb{R}_+^N$, $t_i(s) = s_i \int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v})$, $r_i(s) = 1$ for all $s \in S$ and for all i, and

$$\boldsymbol{\mu}_i(\mathbf{v}|s) = \begin{cases} \frac{\mathbf{1}\{s_i \in \mathbb{M}(s)\}}{|\mathbb{M}(s)|}, & \text{if } s_i \leq v_i \\ 0, & \text{if } s_i > v_i \end{cases},$$

for all i, for all $\mathbf{v} \in V$, and for all $s \in S$, where $\mathbb{M}(s) := \mathrm{argmin}_i\{s_i\}$

As hinted by the many examples above, *any* Bayesian game that models competition among $K \leq N$ firms can be regarded as a market structure. As such, our analyses of market structures apply to all possible static models of competition with fixed preferences and technology, regardless of a model's assumptions about firm conduct, market power,

[5] We thank Fiona Scott Morton for kindly pointing us to this example. See Garthwaite and Morton (2017) for more discussion on PBMs.

price determination, or profit sharing rules.[6] Any dynamic model that can be represented in strategic form is also eligible. Markets in which prices are determined via bilateral bargaining are also eligible.[7]

Of course, not all competitive games among $K \leq N$ firms have an equilibrium. Furthermore, even if an equilibrium exists, some equilibria might be extremely difficult to characterize. A great benefit of our framework is that, as explained below, it bypasses explicit characterizations of equilibria and only focuses on the outcomes.

Among this broad range of market structures, our main interest is to characterize the efficient market structures and explore ways to implement them. To this end, we first formally define our notion of efficiency.

2.4 Defining Efficiency

For any market structure $(S, r, \boldsymbol{\mu}, t)$ and for any Bayes-Nash equilibrium σ of the induced Bayesian game, let

$$\Pi_i(\theta_i|S, r, \boldsymbol{\mu}, t; \sigma) := \mathbb{E}_{\theta_{-i}}\left[\pi_i(\sigma_i(\theta_i), \sigma_{-i}(\theta_{-i}), \theta_i|S, r, \boldsymbol{\mu}, t)\right]$$

denote firm i's interim profit and let

$$\Sigma(S, r, \boldsymbol{\mu}, t; \sigma) := \mathbb{E}_\theta\left[\int_V \sum_{i=1}^N v_i \boldsymbol{\mu}_i(\boldsymbol{v}|\sigma(\theta))F(\mathrm{d}\mathbf{v}) - \sum_{i=1}^N t_i(\sigma(\theta))\right]$$

denote the expected consumer surplus. With this notation, we have the following definitions of dominance and efficiency:

Definition 1. A market structure $(S, r, \boldsymbol{\mu}, t)$ and a Bayes-Nash equilibrium σ *dominates*

[6] Fixed preferences and technology imply that our framework does not suit settings where these two objects are endogenous. Examples of this kind are costly consumer search (e.g., Carlson and McAfee, 1983; Stahl, 1989), informative advertisement (e.g., Butters, 1977; Grossman and Shapiro, 1984; Robert and Stahl, 1993), cost-reducing investment (e.g., Laffont and Tirole, 1986), or network externalities (e.g., Katz and Shapiro, 1985).

[7] Many markets rely crucially on merchant or agent intermediaries (e.g., coffee beans, automobiles, comic books, motion pictures). Intermediaries can readily fit into our framework by adding more players without private information who receive shares of the transfers. We do not explicitly include intermediaries in our model for simplicity.

another market structure $(S', r', \boldsymbol{\mu}', t')$ and Bayes-Nash equilibrium σ' if

$$\Sigma(S, r, \boldsymbol{\mu}, t; \sigma) \geq \Sigma(S', r', \boldsymbol{\mu}', t'; \sigma')$$

and

$$\Pi_i(\theta_i | S, r, \boldsymbol{\mu}, t; \sigma) \geq \Pi_i(\theta_i | S', r', \boldsymbol{\mu}', t'; \sigma')$$

for all i and for all $\theta_i \in \Theta_i$, with at least one inequality being strict.

Definition 2. A market structure $(S, r, \boldsymbol{\mu}, t)$ is *efficient* if there exists a Bayes-Nash equilibrium σ in the Bayesian game induced by $(S, r, \boldsymbol{\mu}, t)$ such that no other market structures and Bayes-Nash equilibria dominate $(S, r, \boldsymbol{\mu}, t)$ and σ.

Remark 1. We are interested in *interim* efficient market structures, as opposed to merely *ex-ante* efficient market structures. This not only produces more general results, but also allows for a crucial implication from a regulatory perspective. Specifically, suppose that there is a designer (i.e., a regulator) who does not have information about the firms' realized costs but is able to intervene in the market to design and enforce market structures. A reasonable criterion for good regulation is that each firm receives a profit no less than what it would have earned under the existing market structure without any intervention. As each firm has private information about its type θ_i, satisfying this criterion requires that each firm's profit must be no less than its *interim* equilibrium payoff in absence of intervention. That exact requirement inhabits our definition of an interim efficient market structure.[8]

Remark 2. In defining efficiency, we account for consumer surplus at the global level across all consumers in aggregate, not at the micro level of an individual consumer in isolation. This perspective suits our interest in the efficient regulation of an entire market rather than the regulation of individual consumer-firm relations. A consequence of our choice is that even under an efficient market structure, a consumer might end up with negative surplus ex post (but never ex ante). While no consumer will ever pay more for a product than her value for it under the efficient market structures we characterize, some consumers could end up in the

[8]Relatedly, Dworczak et al. (forthcoming) also characterizes the interim efficient mechanisms. There, the interim criterion arrives naturally as agents have differentiated marginal utilities for money. While in our setting the interim criterion is due to the desirability of the regulated market from firms' perspectives.

red if taxed. Consumers are granted no outside options in our model, meaning they cannot evade this possibility. Negative ex post individual consumer surplus also is conceivable in Baron and Myerson (1982)—which too regards an entire market's regulation—but not in Myerson and Satterthwaite (1983), where the entire focus is on a single buyer and single seller.[9]

3 Efficiency of PRYCE CAP Market Structures

In what follows, we introduce our main result. As noted in the previous section, our main interest is in characterizing the efficient market structures and exploring practical ways to implement them. Although there are infinitely many possible market structures and some of them can be extremely complex, we show that the efficient market structures are "simple," in the sense that any efficient market structure is equivalent to one that belongs to a natural class. This class of market structures involves price competition with transfers and yardstick price caps, and we refer to this class as PRYCE CAP market structures.

Definition 3. $(S, r, \boldsymbol{\mu}, t)$ is a price competition with transfers and yardstick price caps (PRYCE CAP) market structure if, for any i,

1. $S_i = \mathbb{R}_+$.
2. For any $s \in S$, $r_i(s) = \mathbf{1}\{s_i \leq \bar{p}_i(s_{-i})\}$, for some $\bar{p}_i : S_{-i} \to \mathbb{R}_+ \cup \{\infty\}$.
3. For any $\mathbf{v} \in V$ and for any $s \in S$,

$$\boldsymbol{\mu}_i(\mathbf{v}|s) = \begin{cases} 1, & \text{if } v_i - s_i > \max_{\{j|r_j(s)=1,\, j\neq i\}}(v_j - s_j)^+ \text{ and } r_i(s) = 1 \\ 0, & \text{if } v_i - s_i < \max_{\{j|r_j(s)=1,\, j\neq i\}}(v_j - s_j)^+ \text{ or } r_i(s) = 0 \end{cases}.$$

4. For any $s \in S$, $t_i(s) = s_i \int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(d\mathbf{v}) - \tau_i(s_i)$, for some $\tau_i : S_i \to \mathbb{R}$.

Under a PRYCE CAP market structure,[10] each firm i simultaneously announces a price $s_i \geq 0$. Given the announced prices $s = (s_1, \ldots, s_N)$, a firm is first selected into the market

[9] As a result, the ex post efficient allocation is in fact implementable through a budget-balanced mechanism in our setting.

[10] In naming the price caps, we use the word "yardstick" in a way similar to Shleifer (1985)'s use of the word, in that a firm's regulation depends on characteristics of other firms. In Shleifer (1985), the price a

based on whether its announced price s_i is below its price cap $\bar{p}_i(s_{-i})$. Among the firms that enter the market, consumers then see the announced prices and decide which firm to buy from. Finally, each firm is compensated or taxed via lump-sup transfers from consumers. This transfer amount $\tau_i(s_i)$ depends only on the firm's own price.

Notice that if $\bar{p}_i(s_{-i}) = \infty$ and $\tau_i(s_i) = 0$ for all i and for all s, a PRYCE CAP market structure $(S, r, \boldsymbol{\mu}, t)$ simply reduces to a pure price competition model (see Example 2 above). From this perspective, PRYCE CAP market structures can be regarded as generalizations of pure price competition models that are commonly assumed, with the differences being re-distributional transfers $\{\tau_i\}_{i=1}^N$ and price caps $\{\bar{p}_i\}_{i=1}^N$.

The yardstick price caps embed operating licenses. A firm is granted a license if and only if its announced price is below its yardstick price cap. Hence, the rule for market entry and the price caps are intimately linked. When choosing a price to announce, a firm accounts for both its own price cap and the effect that its choice will have on the price caps of other firms.

With the formal definition of PRYCE CAP market structures presented, we now state our main result.

Theorem 1. *Any efficient market structure is equivalent to a PRYCE CAP market structure.*

The significance of Theorem 1 is that, even though infinitely many market structures exist and some of which might be extremely complex, the efficient market structures are always equivalent to a PRYCE CAP market structure. This means that a designer does not have to search across infinitely many market structures, nor does she have to consider complex market structures to achieve efficiency. Efficiency can be reached by simply enforcing the correct PRYCE CAP market structure.

Furthermore, Theorem 1 also implies that the designer does not have to be omniscient to design an efficient market structure. More precisely, to implement a PRYCE CAP market structure in an existing market, the designer need not know each consumer's product value

monopolist *receives* depends on the costs of other firms in similar markets, so as to encourage the monopolist to reduce costs efficiently. Here, a firm sets its own price, but it faces an upper limit that hinges on the announced prices of competitors in the same market.

$\mathbf{v}$, the manner in which firms compete, the way prices are determined, or the division of surplus.

She also does not need to control every aspect of the market, nor does she need to interfere with transactions between consumers and firms. To achieve an efficient outcome among every possible market structures, a designer need only be capable of enforcing price competition among firms, yardstick price caps, and lump-sum transfers that depend on each firm's public price.

The operating licenses that are embedded in the yardstick price caps are important for reaching social efficiency. No license is granted and entry is restricted if a firm's announced price is above its unique cap. Barred firms can include those that would have otherwise earned positive profits if they had been allowed entry.

The equivalence between interim efficient market structures and PRYCE CAP market structures implies that a designer need not struggle to find ways to improve the structure of a market. If a structure is not already efficient, a designer can always improve the market's efficiency via a price competition structure with correctly specified transfers and yardstick price caps. We summarize this insight in Corollary 1 below.

Corollary 1. *For any market structure $(S, r, \boldsymbol{\mu}, t)$, either one of the following holds:*

1. $(S, r, \boldsymbol{\mu}, t)$ *is efficient.*

2. *There exists a PRYCE CAP market structure that dominates* $(S, r, \boldsymbol{\mu}, t)$*.*

4 Proof of Theorem 1

This section provides the proof of Theorem 1. First, notice that by the revelation principle (Myerson, 1979), it is without loss to restrict attention to incentive compatible and individually rational direct mechanisms. A direct mechanism is a market structure $(S, r, \boldsymbol{\mu}, t)$ where $S_i = \Theta_i$ for all i. For simplicity, we refer to a direct mechanism as a mechanism, and we denote it by $(r, \boldsymbol{\mu}, t)$ hereafter when there is no confusion.

A mechanism is said to be incentive compatible if, for all i and for all $\theta_i, \theta_i' \in \Theta_i$,

$$\begin{aligned}\mathbb{E}_{\theta_{-i}}&\left[t_i(\theta_i,\theta_{-i}) - r_i(\theta_i,\theta_{-i})\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i,\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\\ &\geq \mathbb{E}_{\theta_{-i}}\left[t_i(\theta_i',\theta_{-i}) - r_i(\theta_i',\theta_{-i})\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i',\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right],\end{aligned} \tag{IC}$$

and it is said to be individually rational if, for all $\theta_i \in \Theta_i$,

$$\mathbb{E}_{\theta_{-i}}\left[t_i(\theta_i,\theta_{-i}) - r_i(\theta_i,\theta_{-i})\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i,\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right] \geq 0. \tag{IR}$$

Under any mechanism $(r, \boldsymbol{\mu}, t)$ satisfying (IC) and (IR), firm i's interim expected profit is

$$\Pi_i(\theta_i|r,\boldsymbol{\mu},t) = \mathbb{E}_{\theta_{-i}}\left[t_i(\theta_i) - r_i(\theta)\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right],$$

while the expected consumer surplus is

$$\Sigma(r,\boldsymbol{\mu},t) := \mathbb{E}_\theta\left[\sum_{i=1}^N r_i(\theta)\int_V \boldsymbol{\mu}_i(\mathbf{v}|c)v_i F(\mathrm{d}\mathbf{v}) - \sum_{i=1}^N t_i(\theta)\right].$$

Notice that a mechanism $(r, \boldsymbol{\mu}, t)$ is efficient if and only if it maximizes a weighed sum of the industrial profit (i.e., the sum of firms' interim profits and the expected consumer surplus). Thus, the set of efficient mechanisms is equivalent to the set of solutions to the following class of maximization problem

$$\sup_{(r,\boldsymbol{\mu},t)}\left[\sum_{i=1}^N \int_{\Theta_i} \Pi_i(\theta_i|r,\boldsymbol{\mu},t)\Lambda_i(\mathrm{d}\theta_i) + \Sigma(r,\boldsymbol{\mu},t)\right], \tag{1}$$

subject to (IC) and (IR), where, for all i, Λ_i is a nondecreasing and right-continuous function with $0 \leq \Lambda_i(\theta_i) \leq G_i(\theta_i)$ for all $\theta_i \in \Theta_i$.[11]

Meanwhile, by the envelope theorem, the incentive compatibility constraint (IC) can be characterized by a revenue equivalence formula and a monotonicity condition, as summarized by the following lemma.

[11]Notice that this design problem nests that of Baron and Myerson (1982). Specifically, when $N = 1$ and when $\Lambda_i(\theta_i) = (1-\alpha)G_i(\theta_i)$ for all i and for all θ_i, for some $\alpha \in [0,1]$, the design problem reduces to that of Baron and Myerson (1982).

Lemma 1. *A mechanism $(r, \boldsymbol{\mu}, t)$ is incentive compatible if and only if, for all i, there exists a constant $\bar{t}_i \in \mathbb{R}$ such that*

1. *For any i and for any $\theta_i \in \Theta_i$,*

$$\begin{aligned}&\mathbb{E}_{\theta_{-i}}[t_i(\theta_i, \theta_{-i})]\\ =&\bar{t}_i + \mathbb{E}_{\theta_{-i}}\left[r_i(\theta)\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right) + \int_{\theta_i}^{\bar{\theta}_i} r_i(x, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|x, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\mathrm{d}x\right].\end{aligned}$$

2. *For any i, the function*

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]$$

is nonincreasing.

Proof. See Appendix. ■

By Lemma 1, using integration by parts, for any incentive compatible mechanism $(r, \boldsymbol{\mu}, t)$ and for all i,

$$\begin{aligned}\mathbb{E}_\theta[t_i(\theta)] =& \int_{\Theta_i} \mathbb{E}_{\theta_{-i}}[t_i(\theta_i)]G_i(\mathrm{d}\theta_i)\\ =&\bar{t}_i + \int_{\Theta_i} \theta_i \mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]G_i(\mathrm{d}\theta_i)\\ &+ \int_{\Theta_i} G_i(\theta_i)r_i(\theta_i, \theta_{-i})\mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\mathrm{d}\theta_i.\end{aligned}$$

Thus, expected consumer surplus can be written as

$$\begin{aligned}\Sigma(r, \boldsymbol{\mu}, t) =&\mathbb{E}_\theta\left[\sum_{i=1}^N r_i(\theta_i, \theta_{-i})\int_V v_i \boldsymbol{\mu}_i(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v})\right]\\ &- \sum_{i=1}^N \int_{\Theta_i} \theta_i \mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]G_i(\mathrm{d}\theta_i)\\ &- \sum_{i=1}^N \int_{\Theta_i} G_i(\theta_i)r_i(\theta_i, \theta_{-i})\mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\mathrm{d}\theta_i - \sum_{i=1}^N \bar{t}_i.\end{aligned}$$

Meanwhile, using Lemma 1 and integration by parts, for each firm i,

$$\int_{\Theta_i} \Pi_i(\theta_i|r,\boldsymbol{\mu},t)\Lambda_i(\mathrm{d}\theta_i) = \int_{\Theta_i} \Lambda_i(\theta_i)\mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i,\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i,\theta_{-i})F(\mathrm{d}\mathbf{v})+\kappa_i\right)\right]\mathrm{d}\theta_i + \Lambda_i(\overline{\theta}_i)\bar{t}_i$$

The following lemma further simplifies the above expression.

Lemma 2. *For any i and for any nondecreasing, right-continuous function Λ_i with $0 \leq \Lambda_i(\theta_i) \leq G_i(\theta_i)$, there exists a nondecreasing function $\phi_i^{\Lambda_i} : \Theta_i \to \mathbb{R}_+$ such that*

$$\int_{\Theta_i} \theta_i Q_i(\theta_i)G(\mathrm{d}\theta_i) + \int_{\Theta_i} (G_i(\theta_i)-\Lambda_i(\theta_i))Q_i(\theta_i)\,\mathrm{d}\theta_i \leq \int_{\Theta_i} \phi_i^{\Lambda_i}(\theta_i)Q_i(\theta_i)G_i(\mathrm{d}\theta_i)$$

for any nonincreasing function $Q_i : \Theta_i \to \mathbb{R}_+$, and the equality holds whenever Q_i is measurable with respect to the σ-algebra generated by $\phi_i^{\Lambda_i}$.

Proof. See Appendix. ■

Combining Lemma 1 and Lemma 2, the value of Problem (1) is bounded from above by the solution of

$$\sup_{r,\boldsymbol{\mu}}\left\{\mathbb{E}_\theta\left[\sum_{i=1}^N r_i(\theta)\left(\int_V \left(v_i - \phi_i^{\Lambda_i}(\theta_i)\right)\boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) - \phi_i^{\Lambda_i}(\theta_i)\kappa_i\right)\right] - \sum_{i=1}^N (1-\Lambda_i(\overline{\theta}_i))\bar{t}_i\right\}, \tag{2}$$

subject to

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i,\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i,\theta_{-i})F(\mathrm{d}\mathbf{v})+\kappa_i\right)\right] \text{ is nonincreasing,} \tag{3}$$

By Lemma 1, any individually rational mechanism must have $\bar{t}_i \geq 0$ for all i and thus, it is without loss to set $\bar{t}_i = 0$ for all i.

In what follows, we characterize the solution of Problem (1) by finding a solution to Problem (2) first and then verifying that the objective of Problem (1) equals to the objective of Problem (2) under this solution. To this end, define a market structure $(r^*, \boldsymbol{\mu}^*, t^*)$ as follows: For any $\theta \in \Theta$, let $\mathcal{E}^*(\theta)$ be largest solution (in terms of the strong set order) of

$$\max_{\mathcal{E}\subseteq\{1,\dots,n\}}\left(\int_V \max_{i\in\mathcal{E}}(v_i - \phi_i^{\Lambda_i}(\theta_i))^+F(\mathrm{d}\mathbf{v}) - \sum_{i\in\mathcal{E}}\phi_i^{\Lambda}(\theta_i)\kappa_i\right).$$

Then, let

$$\boldsymbol{\mu}_i^*(\mathbf{v}|\theta) := \begin{cases} \frac{1}{|\mathbb{M}^*(\mathbf{v},\theta)|}, & \text{if } v_i \geq \phi_i^{\Lambda_i}(\theta_i) \text{ and } i \in \mathbb{M}^*(\mathbf{v},\theta) \\ 0, & \text{otherwise} \end{cases},$$

where $\mathbb{M}^*(\mathbf{v},\theta) := \operatorname{argmax}_{j\in\mathcal{E}^*(\theta)}\{v_j - \phi_j^{\Lambda_j}(\theta_j)\}$, for all i, for all $\mathbf{v} \in V$, and for all $\theta \in \Theta$;

$$r_i^*(\theta) = \mathbf{1}\{i \in \mathcal{E}^*(\theta)\}$$

for all i and for all $\theta \in \Theta$; and

$$\begin{aligned} &t_i^*(\theta) = T_i^*(\theta_i) \\ :=&\mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\theta_i\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right) - \int_{\underline{\theta}_i}^{\overline{\theta}_i} r_i^*(x,\theta_{-i})\Bigg(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|x,\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\Bigg)\,\mathrm{d}x\right], \end{aligned}$$

for all i and for all $\theta \in \Theta$.

Lemma 3. *The mechanism* $(r^*, \boldsymbol{\mu}^*, t^*)$ *solves Problem* (2)*. Furthermore,*

$$\sum_{i=1}^N \int_{\Theta_i} \Pi(\theta_i|r^*,\boldsymbol{\mu}^*,t^*)\Lambda_i(\mathrm{d}\theta_i) + \Sigma(r^*,\boldsymbol{\mu}^*,t^*) = \mathbb{E}_\theta\left[\sum_{i=1}^N r_i^*(\theta)\left(\int_V (v_i - \phi_i^{\Lambda_i}(\theta_i))\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) - \phi_i^{\Lambda_i}(\theta_i)\kappa_i\right)\right].$$

Proof. See Appendix ■

Lemma 3 implies that the mechanism $(r^*, \boldsymbol{\mu}^*, t^*)$ is a solution to Problem (1). Furthermore, Lemma 1 and Lemma 2 imply that any other solution of Problem (1) must be equivalent to $(r^*, \boldsymbol{\mu}^*, t^*)$ with probability 1.[12]

Now consider any efficient mechanism, as noted above, it without loss to assume that this mechanism is $(r^*, \boldsymbol{\mu}^*, t^*)$. To see that $(r^*, \boldsymbol{\mu}^*, t^*)$ is equivalent to a PRYCE CAP market structure, consider the market structure $(S, r^{\mathcal{P}}, \boldsymbol{\mu}_i^{\mathcal{P}}, t^{\mathcal{P}})$ as follows: $S_i := \mathbb{R}_+$ for all i;

$$r_i^{\mathcal{P}}(s) := \mathbf{1}\{i \in \mathcal{E}^{\mathcal{P}}(s)\},$$

[12]Except for the tie breaking rules that does not affect efficiency.

for all $s \in S$, where $\mathcal{E}^{\mathcal{P}}(s)$ is the largest solution (in terms of the strong set order) of

$$\max_{\mathcal{E} \subseteq \{1,\dots,n\}} \left(\int_V \max_{i \in \mathcal{E}} (v_i - s_i)^+ F(\mathrm{d}\mathbf{v}) - \sum_{i \in \mathcal{E}} s_i \kappa_i \right),$$

for all $s \in S$;

$$\boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|s) = \begin{cases} \frac{1}{|\mathbb{M}(\mathbf{v},s)|}, & \text{if } v_i \geq s_i \text{ and } i \in \mathbb{M}(\mathbf{v},s) \\ 0, & \text{otherwise} \end{cases},$$

where $\mathbb{M}(\mathbf{v}, s) := \mathrm{argmax}_{j \in \mathcal{E}^{\mathcal{P}}(s)}\{v_j - s_j\}$; and

$$t_i^{\mathcal{P}}(s) := s_i \mathbb{E}_{\theta_{-i}} \left[r_i^{\mathcal{P}}(s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) \int_V \boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) F(\mathrm{d}\mathbf{v}) \right] - \tau_i^*((\phi_i^{\Lambda_i})^{-1}(s_i)),$$

where

$$\tau_i^*(\theta_i) := \phi_i^{\Lambda_i}(\theta_i) \mathbb{E}_{\theta_{-i}} \left[r_i^*(\theta) \int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta) F(\mathrm{d}\mathbf{v}) \right] - T_i^*(\theta_i)$$

and $(\phi_i^{\Lambda_i})^{-1}(s_i) := \inf\{\theta_i \in \Theta_i | \phi_i^{\Lambda_i}(\theta_i) \geq s_i\}$, for all i and for all $s_i \in S_i$.

Notice that for any i, any $s_{-i} \in S_{-i}$ and any $s_i, s_i' \in S_i$, if $s_i > s_i'$ and $i \in \mathcal{E}^{\mathcal{P}}(s_i, s_{-i})$, then it must be that $i \in \mathcal{E}^{\mathcal{P}}(s_i', s_{-i})$. Thus, for any i and for any $s_{-i} \in S_{-i}$, there exists $\bar{p}_i(s_{-i}) \in \mathbb{R}_+ \cup \{\infty\}$ such that $i \in \mathcal{E}^{\mathcal{P}}(s_i, s_{-i})$ if and only if $s_i \leq \bar{p}_i(s_{-i})$. Thus, $(S, r^{\mathcal{P}}, \boldsymbol{\mu}^{\mathcal{P}}, t^{\mathcal{P}})$ is indeed a PRYCE CAP market structure.

The following lemma completes the proof.

Lemma 4. *The PRYCE CAP market structure* $(S, r^{\mathcal{P}}, \boldsymbol{\mu}_i^{\mathcal{P}}, t^{\mathcal{P}})$ *has a Bayes Nash equilibrium* $\sigma^{\mathcal{P}}$ *that induces the same outcome as* $(r^*, \boldsymbol{\mu}^*, t^*)$. *Specifically, for any* i, *for any* $\theta_i \in \Theta_i$, *and for any* $\mathbf{v} \in V$,

$$\boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|\sigma^{\mathcal{P}}(\theta)) = \boldsymbol{\mu}_i^*(\mathbf{v}|\theta); \quad r_i^{\mathcal{P}}(\sigma^{\mathcal{P}}(\theta)) = r_i^*(\theta); \quad \textit{and} \quad t_i^{\mathcal{P}}(\sigma^{\mathcal{P}}(\theta)) = t_i^*(\theta).$$

Proof. See Appendix ■

5 PRYCE CAP Example and *Laissez-Faire*

Here we present a concrete example to illustrate how firm choices affect the yardstick price caps and the entry rules in a PRYCE CAP market structure. We also introduce *laissez-faire* market structures to study whether and when they are socially efficient.

5.1 Example

Suppose that the number of potentially active firms $N = 2$ and that consumer values and firm types $v_1, \theta_1, v_2, \theta_2 \in [0, 1]$ are independently drawn from a uniform distribution. Suppose further that the commonly known fixed cost parameters $\kappa_1 = \kappa_2 = 1$ and that the Pareto weight functions $\Lambda_1(x) = \Lambda_2(x) = x$ for all $x \in [0, 1]$. Then, the following three polynomials determine the yardstick price caps $\bar{p}_1, \bar{p}_2$ and the set of firms granted operating licenses $\mathcal{E}^{\mathcal{P}}$. The values s_1 and s_2 denote the reported strategies of firm 1 and firm 2, which, under a PRYCE CAP market structure, are their published prices.

$$\Sigma_{\{1,2\}}(s_1, s_2) := \begin{cases} \frac{(1-s_1)^3+(1-s_2)^3}{3} + \frac{s_1(1-s_2)^2+s_2(1-s_1)^2-2(s_1+s_2)}{2}, & \text{if } (s_1, s_2) \in [0,1]^2 \\ 0, & \text{otherwise} \end{cases};$$

and

$$\Sigma_{\{1\}}(s_1, s_2) := \begin{cases} \frac{(1-s_1)^2-2s_1}{2}, & \text{if } s_1 \in [0,1] \\ 0, & \text{otherwise} \end{cases}; \quad \Sigma_{\{2\}}(s_1, s_2) := \begin{cases} \frac{(1-s_2)^2-2s_2}{2}, & \text{if } s_2 \in [0,1] \\ 0, & \text{otherwise} \end{cases},$$

for all $(s_1, s_2) \in \mathbb{R}^2_+$. It then follows that

$$\mathcal{E}^{\mathcal{P}}(s) = \begin{cases} \{1,2\}, & \text{if } \Sigma_{\{1,2\}}(s) \geq \max\{\Sigma_{\{1\}}(s), \Sigma_{\{2\}}(s), 0\} \\ \{1\}, & \text{if } \Sigma_{\{1\}}(s) > \Sigma_{\{1,2\}}(s) \text{ and } \Sigma_{\{1\}}(s) \geq \max\{\Sigma_{\{2\}}(s), 0\} \\ \{2\}, & \text{if } \Sigma_{\{2\}}(s) > \max\{\Sigma_{\{1,2\}}(s), \Sigma_{\{1\}}(s)\} \text{ and } \Sigma_{\{2\}}(s) \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

The set-valued function $\mathcal{E}^{\mathcal{P}}(\cdot)$ and the price caps $\bar{p}_1$, $\bar{p}_2$ are depicted in Figure 1.

Figure 1: Values of $\mathcal{E}^{\mathcal{P}}$

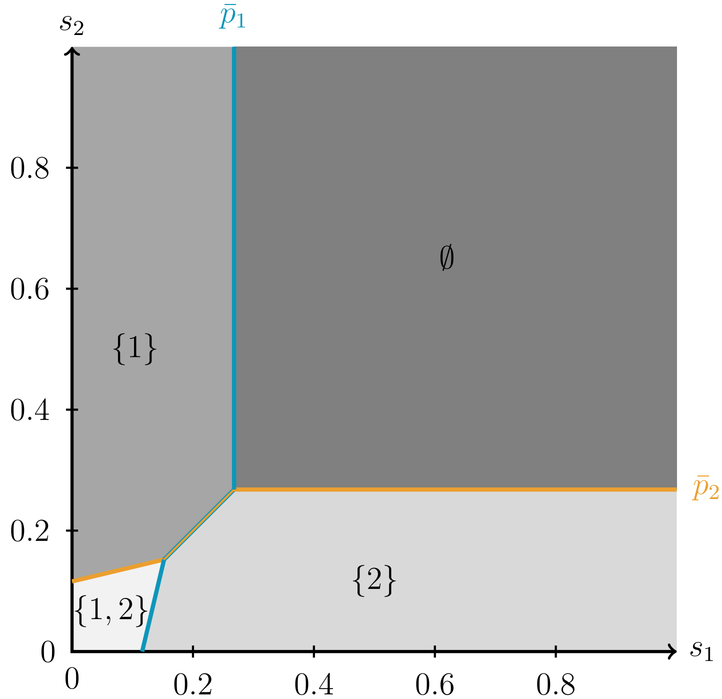

Figure 1 illustrates the tight link between the yardstick price caps and the sets of firms optimally granted operating licenses. In the figure, the set of (undominated) prices $[0,1]^2$ is partitioned into four regions, where each region of (s_1, s_2) is mapped into different values of $\mathcal{E}^{\mathcal{P}}(s_1, s_2)$. As a result, the boundaries of the regions define the yardstick price caps. The gold-orange curve represents firm 2's price cap $\bar{p}_2$ as a function of s_1, and the blue-green curve represents firm 1's price cap $\bar{p}_1$ as a function of s_2. Given firm 1's published price s_1, firm 2 is excluded from the market if it posts a price $s_2 > \bar{p}_2(s_1)$. Similarly, given firm 2's published price s_2, firm 1 is excluded from the market if it posts a price $s_1 > \bar{p}_1(s_2)$.

5.2 When is the *Laissez-Faire* Outcome Efficient?

A crucial feature of PRYCE CAP market structures is that transfers independent of transactions between consumers and firms exist. From a designer's perspective, this means that implementation of any efficient market structure entail interventions in the form of taxation or subsidy, even if firms are already engaging in price competition.

A natural question then arises: whether and when is it possible for a market structure without any intervention to be efficient? A simple example is the well-known Bertrand model with complete information, homogeneous preferences, and no fixed costs. In this case, there exists an equilibrium in which every firm prices at the lowest marginal cost and all consumers buy from the firm with the lowest marginal cost, inducing an efficient outcome even without any form of intervention. Nevertheless, Theorem 2 below shows that this example is rather non-generic, in the sense that any (non-degenerate) *laissez-faire* market structure is efficient only if there is complete information.

To simplify the statement of the theorem, we further introduce some assumptions about the value distribution F. For the rest of this subsection, assume that F has full support and is absolutely continuous on V, and that $v_i \geq \underline{\theta}_i$ for all i with positive F-measure. We now formally define a *laissez-faire* market structure.

Definition 4. $(S, r, \boldsymbol{\mu}, t)$ is a *laissez-faire* market structure if, for all i, there exists $\boldsymbol{p}_i : S \to \mathbb{R}_+$ such that

$$t_i(s) = r_i(s)\boldsymbol{p}_i(s) \int_V \boldsymbol{\mu}_i(\mathbf{v}|s) F(\mathrm{d}\mathbf{v}),$$

for all i and for all $s \in S$; and that

$$(\boldsymbol{p}_i(s) - v_i)^+ \boldsymbol{\mu}_i(\mathbf{v}|s) = 0,$$

for all i, for all $\mathbf{v} \in V$, and for all $s \in S$.

In other words, under a *laissze-faire* market structure $(S, r, \boldsymbol{\mu}, t)$, a strategy profile s leads to prices $\{\boldsymbol{p}_i(s)\}_{i=1}^N$ for each firm's product. The revenue of firm i, conditional on participating in the market, is then determined by $\boldsymbol{p}_i(s)$ times quantity. Transfers in any other forms are not allowed. Meanwhile, consumers' purchase decisions must be sequentially rational. That is, for any consumer with value vector $\mathbf{v}$, if this consumer buys from firm i when the firms' strategy profile is s (i.e., $\boldsymbol{\mu}_i(\mathbf{v}|s) > 0$)), it must be that firm i's price is below this consumer's value for firm i's product (i.e., $v_i \geq \boldsymbol{p}_i(s)$). This definition reflects market structures in which firms and consumers interact without any third-party interference. Therefore, understanding whether it is possible for a *laissez-faire* market structure to be efficient has implications for whether regulation is necessary. From this regard, the following theorem suggests that regulation is generically necessary.

Theorem 2. *A* laissez-faire *market structure is efficient only if* $\text{supp}(G_i) = \{\underline{\theta}_i\}$ *for all* i.

Theorem 2 implies that the only case for a *laissez-faire* market structure to be efficient is when there is no private information. Few strategic settings in theory and arguably none in reality satisfy this requirement. For example, consider Bertrand price competition. While it is well-known that a Bertrand price competition model with complete information has an efficient equilibrium where each firm charges the lowest marginal cost, this example is non-generic. With a small perturbation to firms' information structure, such a *laissez-faire* market structure can no longer be efficient and can be improved by a designer's intervention.

6 Conclusions

This paper proposes a framework to design the structure of an industrial market with the purpose of achieving some social goal. We show that in the search for an efficient market structure among infinitely many, a designer may rely on a simple class that can be implemented without knowledge of individual consumer preferences, realized firm cost, or firm conduct (i.e., the strategic manner in which firms compete). This important class of market structures is regulated price competition, where the regulatory tools are transfers plus yardstick price caps. We refer to this class as PRYCE CAP market structures. Embedded in the yardstick price caps are operating license rights. Each component of this structure has history in regulatory practice, though this paper establishes their special combination. We show that for any existing market structure, either it already is interim Pareto efficient, or a PRYCE CAP market structure Pareto dominates it. Finally, we prove that it is never possible for a market structure to be efficient without intervention, unless there is no private information.

To implement a PRYCE CAP market structure, a designer, we presume, has power to verify and enforce competition exclusively on price, regardless of the kinds of complex, competitive conduct that are common to the industry. But sustaining price competition is not the unique way to achieve efficiency. For certain markets, a clever selection of transfers alone might convert an existing inefficient market structure into an efficient one. Administering such creative transfer schemes would likely be intractable, and the designer would need unearthly knowledge of the competitive game that firms engage. A pivotal contribution of PRYCE CAP market structures is that they require no such awareness, they are simple, and they apply to a broad range of potential firm conducts.

Hence, if the designer can verify and enforce price competition, the search for efficiency ends with PRYCE CAP market structures. In practice, though, a designer might lack such powers entirely or wield them imperfectly. A natural implication of our results is that social efficiency is more easily achieved in industries where a regulator can plausibly uphold price competition. Or rather, more realistically, industries where posting prices *already* drives the nature of competition are better candidates to become efficient.

Our framework has limitations that leaves room for continuing work. First, consumers in our setting have unit demand. A consumer will be a customer to one firm only and will purchase only a single quantity of that firm's product. This assumption is easily defensible for many markets (e.g., automobiles, airplanes, household appliances). The assumption is also widely adopted across models of industrial competition, particularly models of discrete choice that form the backbone of modern empirical industrial organization (McFadden, 1973; Berry et al., 1995). However, the unit demand assumption is special and might be unsuitable for markets in which either the intensive margin matters (e.g., mortgage loans) or consumers regularly purchase multiple differentiated products (e.g., Granny Smith and Honey Crisp apples).

Second, each firm produces a single type of good. In practice, one firm might create more than one product in the same market (e.g., regular and premium version), or one firm might sell products across distinct markets. In developed economies, these kinds of multi-product firms are abundant from horizontal integration. For example, the Clorox Company has a footprint in the markets for disinfecting wipes (Clorox wipes), food dressing (Hidden Valley Ranch), personal skin care (Burt's Bees), water filtration (Brita), and charcoal briquettes (Kingsford), to name a few. Designing market structures for horizontally-integrated firms would encompass several markets simultaneously, and it would need to account for, among several things, how cross-price elasticities among products in a conglomerate's portfolio affects its decision. Market structure design for these types of firms has profound relevance for antitrust issues and is a fascinating avenue to explore.

Finally, the firms in our setting have constant marginal costs and outside options worth zero. All firms have compelling reasons to participate in the designer's mechanism rather than voluntarily escape it as long as their net profits are nonnegative. This assumption simplifies the setting, but it might be less suitable in markets that experience regulatory arbitrage (e.g., commercial banking). The assumption can be relaxed by introducing general, type-dependent outside options. The model would become more challenging to solve, but inserting this change can produce several new insights. Likewise, constant marginal cost technology is a simple and standard assumption to employ, which is why we use it. Incorporating more complex, nonlinear cost functions or capacity choice is a promising research area left for the future.

References

Amador, M. and K. Bagwell (2021): "Regulating a monopolist with uncertain costs without transfers," Working paper, Stanford University, Stanford, CA.

Armstrong, M. (2000): "Optimal multi-object auctions," *Review of Economic Studies*, 67, 455–481.

Armstrong, M. and D. E. Sappington (2004): "Toward a synthesis of models of regulatory policy design with limited information," *Journal of Regulatory Economics*, 26, 5–21.

——— (2006): "Regulation, competition and liberalization," *Journal of Economic Literature*, 44, 325–366.

Armstrong, M. and J. Vickers (2019): "Discriminating against captive customers," *American Economic Review: Insights*, 1, 257–72.

Auriol, E. and J.-J. Laffont (1992): "Regulation by duopoly," *Journal of Economics & Management Strategy*, 1, 507–533.

Baron, D. P. and D. Besanko (1992): "Information, control, and organizational structure," *Journal of Economics & Management Strategy*, 1, 237–275.

Baron, D. P. and R. B. Myerson (1982): "Regulating a Monopolist with Unknown Cost," *Econometrica*, 50, 911–930.

Baumol, W. J. (1982): "Contestable Markets: An Uprising in the Theory of Industry Structure," *American Economic Review*, 72, 1–15.

Berry, S., M. Gaynor, and F. Scott Morton (2019): "Do increasing markups matter? Lessons from empirical industrial organization," *Journal of Economic Perspectives*, 33, 44–68.

Berry, S., J. Levinsohn, and A. Pakes (1995): "Automobile prices in market equilibrium," *Econometrica*, 841–890.

Bertrand, J. (1883): "Book review of theorie mathematique de la richesse sociale and of recherches sur les principles mathematiques de la theorie des richessesj," *Journal de Savants*, 67, 1067–1082.

Butters, G. (1977): "Equilibrium distribution of prices and advertising in oligopoly," *Review of Economic Studies*, 44, 465–491.

Carlson, J. A. and R. P. McAfee (1983): "Discrete equilibrium price dispersion," *Journal of Political Economy*, 91, 480–493.

Che, Y.-K. and I. Gale (2003): "Optimal design of research contests," *American Economic Review*, 93, 646–671.

Cournot, A. (1838): *Recherches sur les principes de la théorie des richesses*, Paris: Hachette.

Dana Jr, J. D. (1993): "The organization and scope of agents: Regulating multiproduct industries," *Journal of Economic Theory*, 59, 288–310.

Demsetz, H. (1968): "Why regulate utilities?" *Journal of Law and Economics*, 11, 55–65.

Dworczak, P., S. D. Kominers, and M. Akbarpour (forthcoming): "Redistribution through Markets," *Econometrica*.

Fullerton, R. L. and R. P. McAfee (1999): "Auctioning entry into tournaments," *Journal of Political Economy*, 107, 573–605.

Garthwaite, C. and F. S. Morton (2017): "Perverse Market Incentives Encourage High Prescription Drug Prices," *ProMarket Blog, Stigler Center at The University of Chicago Booth School of Business*.

Gilbert, R. J. and M. H. Riordan (1995): "Regulating complementary products: A comparative institutional analysis," *RAND Journal of Economics*, 243–256.

Grossman, G. M. and C. Shapiro (1984): "Informative advertising with differentiated products," *Review of Economic Studies*, 51, 63–81.

Guesnerie, R. and J.-J. Laffont (1984): "A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm," *Journal of Public Economics*, 25, 329–369.

Guo, Y. and E. Shmaya (2019): "Robust Monopoly Regulation," Working paper, Northwestern University, Chicago, IL.

Hoppe, H. C., P. Jehiel, and B. Moldovanu (2006): "License auctions and market structure," *Journal of Economics & Management Strategy*, 15, 371–396.

Hotelling, H. (1929): "Stability in Competition," *Economic Journal*, 39, 41–57.

Katz, M. L. and C. Shapiro (1985): "Network externalities, competition, and compatibility," *American Economic Review*, 75, 424–440.

Kjerstad, E. and S. Vagstad (2000): "Procurement auctions with entry of bidders," *International Journal of Industrial Organization*, 18, 1243–1257.

Laffont, J.-J. and J. Tirole (1986): "Using cost observation to regulate firms," *Journal of Political Economy*, 94, 614–641.

——— (1987): "Auctioning incentive contracts," *Journal of Political Economy*, 95, 921–937.

Lewis, T. R. and D. E. Sappington (1988): "Regulating a monopolist with unknown demand," *American Economic Review*, 986–998.

——— (1989): "Regulatory options and price-cap regulation," *RAND Journal of Economics*, 405–416.

Mankiw, N. G. and M. D. Whinston (1986): "Free entry and social inefficiency," *RAND Journal of Economics*, 48–58.

McFadden, D. (1973): *Conditional logit analysis of qualitative choice behavior*, New York: Academic Press, chap. 4, 105–142.

McMillan, J. (1994): "Selling spectrum rights," *Journal of Economic Perspectives*, 8, 145–162.

McNulty, P. J. (1968): "Economic theory and the meaning of competition," *Quarterly Journal of Economics*, 639–656.

Milgrom, P. (1998): "Game theory and the spectrum auctions," *European Economic Review*, 42, 771–778.

Milgrom, P. and I. Segal (2002): "Envelope theorems for arbitrary choice sets," *Econometrica*, 70, 583–601.

Monteiro, P. K. and B. F. Svaiter (2010): "Optimal auction with a general distribution: Virtual valuation without densities," *Journal of Mathematical Economics*, 46, 21–31.

Myerson, R. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47, 61–73.

Myerson, R. B. and M. A. Satterthwaite (1983): "Efficient mechanisms for bilateral trading," *Journal of Economic Theory*, 29, 265–281.

Narasimhan, C. (1988): "Competitive promotional strategies," *Journal of Business*, 427–449.

Palfrey, T. R. (1983): "Bundling decisions by a multiproduct monopolist with incomplete information," *Econometrica*, 463–483.

Perloff, J. M. and S. C. Salop (1985): "Equilibrium with product differentiation," *Review of Economic Studies*, 52, 107–120.

Rey, P. and J. Tirole (2019): "Price caps as welfare-enhancing coopetition," *Journal of Political Economy*, 127, 3018–3069.

Riordan, M. H. and D. E. Sappington (1987): "Awarding monopoly franchises," *American Economic Review*, 375–387.

Robert, J. and D. O. Stahl (1993): "Informative price advertising in a sequential search model," *Econometrica*, 657–686.

Salop, S. C. (1979): "Monopolistic competition with outside goods," *Bell Journal of Economics*, 141–156.

Schmalensee, R. (1989): "Good regulatory regimes," *RAND Journal of Economics*, 417–436.

Severinov, S. (2003): "Optimal organization: Centralization, decentralization or delegation?" Unpublished working paper. Duke University, Durham, NC.

Shleifer, A. (1985): "A theory of yardstick competition," *RAND Journal of Economics*, 319–327.

Stahl, D. O. (1989): "Oligopolistic pricing with sequential consumer search," *The American Economic Review*, 700–712.

Stigler, G. J. (1957): "Perfect competition, historically contemplated," *Journal of Political Economy*, 65, 1–17.

Taylor, C. R. (1995): "Digging for golden carrots: An analysis of research tournaments," *American Economic Review*, 872–890.

Varian, H. R. (1980): "A model of sales," *American Economic Review*, 70, 651–659.

Vickers, J. (1995): "Concepts of competition," *Oxford Economic Papers*, 1–23.

Von Stackelberg, H. (1934): *Market structure and equilibrium*, Springer.

A Proofs

A.1 Proof of Lemma 1

Proof. For necessity, given any incentive compatible mechanism $(r, \boldsymbol{\mu}, t)$, for any i and for any θ_i, θ_i', let

$$u_i(\theta_i, \theta_i') := \mathbb{E}_{\theta_{-i}}\left[t_i(\theta_i', \theta_{-i}) - r_i(\theta_i', \theta_{-i})\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i', \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right].$$

Incentive compatibility implies that

$$U_i(\theta_i) := u_i(\theta_i, \theta_i) = \max_{\theta_i' \in \Theta_i} u_i(\theta_i, \theta_i')$$

for all $\theta_i \in \Theta_i$. Furthermore, since the function $\theta_i \mapsto u_i(\theta_i, \theta_i')$ is affine (and hence absolutely continuous and has a uniformly bounded almost-everywhere derivative) for all $\theta_i' \in \Theta_i$, the envelope theorem (Milgrom and Segal, 2002) implies that U_i is absolutely continuous and the almost-everywhere derivative is

$$U_i'(\theta_i) = \frac{\partial}{\partial c_i} u_i(\theta_i, \theta_i')\bigg|_{\theta_i' = \theta_i} = -\mathbb{E}_{\theta_{-i}}\left[r_i(\theta)\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]$$

and hence, by the fundamental theorem of calculus,

$$U_i(\theta_i) = U_i(\overline{\theta}_i) + \int_{\theta_i}^{\overline{\theta}_i} \mathbb{E}_{\theta_{-i}}\left[r_i(x, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|x, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\mathrm{d}x.$$

Let $\bar{t}_i := -U_i(\overline{\theta}_i)$ and rearrange to get

$$\begin{aligned} &\mathbb{E}_{\theta_{-i}}[t_i(\theta_i, \theta_{-i})] \\ =&\bar{t}_i + \mathbb{E}_{\theta_{-i}}\left[r_i(\theta)\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right) + \int_{\theta_i}^{\overline{\theta}_i} r_i(x, \theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|x, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\mathrm{d}x\right]. \end{aligned}$$

This proves 1. For 2, notice that since $u_i(\cdot, \theta_i')$ is affine, $U_i(\theta_i)$ is a pointwise maximum of a family of affine functions for all θ_i and hence, U_i is convex, which in turn implies that its derivative is nondecreasing. This proves 2.

For sufficiency, consider any mechanism $(r, \boldsymbol{\mu}, t)$ satisfying 1 and 2. Then, for any i and for any $\theta_i, \theta_i' \in \Theta_i$,

$$
\begin{aligned}
&\mathbb{E}_{\theta_{-i}}\left[t_i(\theta_i',\theta_{-i}) - r_i(\theta_i',\theta_{-i})\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i',\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\\
=&\mathbb{E}_{\theta_{-i}}\left[t_i(\theta_i',\theta_{-i}) - r_i(\theta_i',\theta_{-i})\theta_i'\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i',\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\\
&+(\theta_i'-\theta_i)\mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i',\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i',\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\\
=&\int_{\theta_i'}^1 \mathbb{E}_{\theta_{-i}}\left[r_i(x,\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|x,\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\mathrm{d}x - \bar{t}_i\\
&+\int_{\theta_i}^{\theta_i'} \mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i',\theta_{-i})\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i',\theta_{-i})F(\mathrm{d}\mathbf{v})\right]\mathrm{d}x\\
=&\int_{\theta_i}^1 \mathbb{E}_{\theta_{-i}}\left[r_i(x,\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|x,c_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\mathrm{d}x - \bar{t}_i\\
&+\int_{\theta_i}^{\theta_i'}\left(\mathbb{E}_{\theta_{-i}}\left[r_i(\theta_i',\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|c_i',c_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right) - r_i(x,\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|x,c_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\right)\mathrm{d}x\\
\le&\int_{\theta_i}^1 \mathbb{E}_{\theta_{-i}}\left[r_i(x,\theta_{-i})\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|x,c_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]\mathrm{d}x\\
=&\mathbb{E}_{\theta_{-i}}\left[t_i(\theta_i) - r_i(\theta)\theta_i\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) - \theta_i\kappa_i\right)\right],
\end{aligned}
$$

where the second and the last equality follows from condition 1, and the inequality follows from condition 2. Therefore, $(r, \boldsymbol{\mu}, t)$ is incentive compatible. This completes the proof. ■

A.2 Proof of Lemma 2

Proof. For any i, define a measure ν_i on $\mathbb{R}_+$ as

$$\nu_i(A) := \int_A \theta_i G_i(\mathrm{d}\theta_i) + \int_A (G_i(\theta_i) - \Lambda_i(\theta_i))\,\mathrm{d}\theta_i$$

and let $H_i(\theta_i) := \nu_i([0,\theta_i])$. The assertion then follows from Theorem 2 and Theorem 3 of Monteiro and Svaiter (2010). ■

A.3 Proof of Lemma 3

Proof. We first show that for all i,

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta_i,\theta_{-i})\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta_i,\theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]$$

is nonincreasing. To see this, notice that for any i and for any $\theta \in \Theta$,

$$\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) = \int_V \mathbf{1}\{\phi_i^{\Lambda_i}(\theta_i) \le \phi_i^{\Lambda_i}(\theta_j) + v_i - v_j,\ \forall j \in \mathcal{R}^*(\theta),\ j \neq i\}F(\mathrm{d}\mathbf{v}). \tag{4}$$

Moreover, notice that for any i, for any $\theta_{-i} \in \Theta_{-i}$, and for any $\theta_i, \theta_i' \in \Theta_i$ with $\theta_i' < \theta_i$, $i \in \mathcal{R}^*(\theta_i, \theta_{-i})$ implies $i \in \mathcal{R}^*(\theta_i', \theta_{-i})$. Together with the fact that $\phi_i^{\Lambda_i}$ is nondecreasing, it then follows that both (4) and r_i^* are nonincreasing functions of θ_i for all $\theta_{-i} \in \Theta_{-i}$. Therefore,

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]$$

is indeed nonincreasing.

Furthermore, by definition of $(r^*, \boldsymbol{\mu}^*)$, for any $(r, \boldsymbol{\mu})$ such that

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}}\left[r_i(\theta)\left(\int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]$$

is nonincreasing, it must be that

$$\begin{aligned}
&\mathbb{E}_\theta\left[\sum_{i=1}^N r_i(\theta)\left(\int_V (v_i - \phi_i^{\Lambda_i}(\theta_i))\boldsymbol{\mu}_i(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) - \phi_i^{\Lambda_i}(\theta_i)\kappa_i\right)\right] \\
\leq &\mathbb{E}_\theta\left[\sum_{i=1}^N r_i^*(\theta)\left(\int_V (v_i - \phi_i^{\Lambda_i}(\theta_i))\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) - \phi_i^{\Lambda_i}(\theta_i)\kappa_i\right)\right].
\end{aligned}$$

Thus, $(r^*, \boldsymbol{\mu}^*)$ is a solution to (2).

Lastly, by Lemma 2, since the function

$$\theta_i \mapsto \mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right]$$

is nonincreasing and is measurable with respect to $\phi_i^{\Lambda_i}$ for all i, we have

$$\begin{aligned}
&\int_{\Theta_i} \phi_i^{\Lambda_i}(\theta_i)\mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right] G_i(\mathrm{d}\theta_i) \\
=&\int_{\Theta_i} \theta_i\mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right] G_i(\mathrm{d}\theta_i) \\
&+ \int_{\Theta_i} (G_i(\theta_i) - \Lambda_i(\theta_i))\mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right] \mathrm{d}\theta_i
\end{aligned}$$

Therefore,

$$
\begin{aligned}
&\mathbb{E}_\theta\left[\sum_{i=1}^N r_i^*(\theta)\left(\int_V (v_i-\phi_i^{\Lambda_i}(\theta_i))\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v})-\phi_i^{\Lambda_i}(\theta_i)\kappa_i\right)\right]\\
=&\mathbb{E}_\theta\left[\sum_{i=1}^N r_i^*(\theta)\int_V v_i\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v})\right]-\sum_{i=1}^N\left\{\int_{\Theta_i}\phi_i^{\Lambda_i}(\theta_i)\mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v})+\kappa_i\right)\right]G_i(\mathrm{d}\theta_i)\right\}\\
=&\mathbb{E}_\theta\left[\sum_{i=1}^N r_i^*(\theta)\int_V v_i\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v})\right]-\int_{\Theta_i}\theta_i\mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v})+\kappa_i\right)\right]G_i(\mathrm{d}\theta_i)\\
&-\int_{\Theta_i}(G_i(\theta_i)-\Lambda_i(\theta_i))\mathbb{E}_{\theta_{-i}}\left[r_i^*(\theta)\left(\int_V\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v})+\kappa_i\right)\right]\mathrm{d}\theta_i\\
=&\Sigma(r^*,\boldsymbol{\mu}^*,t^*)+\sum_{i=1}^N\int_{\Theta_i}\Pi(\theta_i|r^*,\boldsymbol{\mu}^*,t^*)\Lambda_i(\mathrm{d}\theta_i),
\end{aligned}
$$

as desired. ■

A.4 Proof of Lemma 4

Proof. Consider the mechanism $(S, r^{\mathcal{P}}, \boldsymbol{\mu}_i^{\mathcal{P}}, t^{\mathcal{P}})$. First, notice that by definition of $r^{\mathcal{P}}$ and $\boldsymbol{\mu}_i^{\mathcal{P}}$, for all $\theta\in\Theta$ and for all i,

$$r_i^{\mathcal{P}}(\phi_1^{\Lambda_1}(\theta_1),\dots,\phi_N^{\Lambda_N}(\theta_N))=r_i^*(\theta_1,\dots,\theta_N);$$

and

$$\boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|\phi_1^{\Lambda_1}(\theta_1),\dots,\phi_N^{\Lambda_N}(\theta_N))=\boldsymbol{\mu}_i^*(\mathbf{v}|\theta_1,\dots,\theta_N),$$

for all $\mathbf{v}\in V$. Moreover, by Lemma 1, for each i and for any interval $[\theta_i^1,\theta_i^2]$ on which $\phi_i^{\Lambda_i}$ is constant, T_i^* is also constant. Therefore, for any i and for any $\theta_i\in\Theta_i$, if θ_i belongs to an interval $[\theta_i^1,\theta_i^2]$ on which $\phi_i^{\Lambda_i}$ is a constant, then $(\phi_i^{\Lambda_i})^{-1}(\phi_i^{\Lambda_i}(\theta_i))=\theta_i^2=\theta_i$, Thus, for any i and for any $\theta\in\Theta$,

$$
\begin{aligned}
t_i^{\mathcal{P}}(\phi_1^{\Lambda_1}(\theta_1),\dots,\phi_N^{\Lambda_N}(\theta_N))=&\mathbb{E}_{\theta_{-i}}\left[\phi_i^{\Lambda_i}(\theta_i)r_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i),\phi_{-i}^{\Lambda_{-i}}\theta_{-i})\int_V\boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|\phi_i^{\Lambda_i}(\theta_i),\phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))F(\mathrm{d}\mathbf{v})\right]-\tau_i^*(\theta_i)\\
=&\mathbb{E}_{\theta_{-i}}\left[\phi_i^{\Lambda_i}(\theta_i)r_i^*(\theta)\int_V\boldsymbol{\mu}_i^*(\mathbf{v}|\theta)F(\mathrm{d}\mathbf{v})\right]-\tau_i^*(\theta_i)\\
=&T_i^*(\theta_i)\\
=&t_i^*(\theta_1,\dots,\theta_N),
\end{aligned}
$$

where $\phi_{-i}^{\Lambda_{-i}}:=(\phi_1^{\Lambda_1},\dots,\phi_{i-1}^{\Lambda_{i-1}},\phi_{i+1}^{\Lambda_{i+1}},\dots,\phi_N^{\Lambda_N})$.

It then remains to show that the strategy profile where each firm i with type θ_i chooses $\phi_i^{\Lambda_i}$ is a Bayes-Nash equilibrium in the game induced by $(S, r^{\mathcal{P}}, \boldsymbol{\mu}_i^{\mathcal{P}}, t^{\mathcal{P}})$. Indeed, for any firm i, any type $\theta_i\in\Theta_i$ and for any $s_i\in\phi_i^{\Lambda_i}(\Theta_i)$, given that all other firms follow the strategy

$\phi_{-i}^{\Lambda_{-i}} = (\phi_1^{\Lambda_1}, \ldots, \phi_{i-1}^{\Lambda_{i-1}}, \phi_{i+1}^{\Lambda_{i+1}}, \ldots, \phi_N^{\Lambda_N})$, let $\theta_i' \in \Theta_i$ be such that $\phi_i^{\Lambda_i}(\theta_i') = s_i$. We then have

$$\begin{aligned}
&\mathbb{E}_{\theta_{-i}}\left[t_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i\left(\int_V \boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right] \\
=&\mathbb{E}_{\theta_{-i}}\left[t_i^*(\theta_i, \theta_{-i}) - r_i^*(\theta_i, \theta_{-i})\theta_i\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta_i, \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right] \\
\geq&\mathbb{E}_{\theta_{-i}}\left[t_i^*(\theta_i', \theta_{-i}) - r_i^*(\theta_i', \theta_{-i})\theta_i\left(\int_V \boldsymbol{\mu}_i^*(\mathbf{v}|\theta_i', \theta_{-i})F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right] \\
=&\mathbb{E}_{\theta_{-i}}\left[t_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i'), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i'), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i\left(\int_V \boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|\phi_i^{\Lambda_i}(\theta_i'), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right. \\
&\mathbb{E}_{\theta_{-i}}\left[t_i^{\mathcal{P}}(s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^{\mathcal{P}}(s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i\left(\int_V \boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right.,
\end{aligned}$$

where the inequality follows from the fact that $(r^*, \boldsymbol{\mu}^*, t^*)$ is incentive compatible. Meanwhile, it is easy to verify that for any firm i, any type $\theta_i \in \Theta_i$, and for any $s_i \notin \phi_i^{\Lambda_i}(\Theta_i)$, given that all other firms follow the strategy $\phi_{-i}^{\Lambda_{-i}}$,

$$\begin{aligned}
&\mathbb{E}_{\theta_{-i}}\left[t_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i'), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i\left(\int_V \boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|\phi_i^{\Lambda_i}(\theta_i), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right. \\
\geq&\mathbb{E}_{\theta_{-i}}\left[t_i^{\mathcal{P}}(\phi_i^{\Lambda_i}(\theta_i'), \phi_{-i}^{\Lambda_{-i}}(\theta_{-i})) - r_i^{\mathcal{P}}(s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))\theta_i\left(\int_V \boldsymbol{\mu}_i^{\mathcal{P}}(\mathbf{v}|s_i, \phi_{-i}^{\Lambda_{-i}}(\theta_{-i}))F(\mathrm{d}\mathbf{v}) + \kappa_i\right)\right..
\end{aligned}$$

Together, it then follows that $(\phi_1^{\Lambda_1}, \ldots, \phi_N^{\Lambda_N})$ is indeed a Bayes-Nash equilibrium in the game induced by $(S, r^{\mathcal{P}}, \boldsymbol{\mu}_i^{\mathcal{P}}, t^{\mathcal{P}})$. This completes the proof. ■

A.5 Proof of Theorem 2

Proof. Consider any *laissez-faire* market structure. By the revelation principle and by Lemma 1, it is without loss to assume that $S = \Theta$ and that it satisfies conditions 1 and 2 of Lemma 1. Denote this mechanism as $(r, \boldsymbol{\mu}, \boldsymbol{p})$. Suppose that this mechanism is efficient. Then there exists nondecreasing functions $\{\Lambda_i\}_{i=1}^N$, with $0 \leq \Lambda_i(\theta_i) \leq G_i(\theta_i)$ for all i and for all $\theta_i \geq 0$, and set valued functions $\mathcal{E} : \Theta \to 2^{\{1,\ldots,n\}}$ with

$$\mathcal{E}(\theta) \in \operatorname*{argmax}_{\mathcal{E} \subseteq \{1,\ldots,n\}} \left(\int_V \max_{i \in \mathcal{E}} (v_i - \phi^{\Lambda_i}(\theta_i))^+ F(\mathrm{d}\mathbf{v}) - \sum_{i \in \mathcal{E}} s_i \kappa_i\right),$$

for all $\theta \in \Theta$, such that for almost all $\mathbf{v}$ and for G-almost all $\theta \in \Theta$,

$$\boldsymbol{\mu}_i(\mathbf{v}|\theta) = \begin{cases} 1, & \text{if } v_i - \phi_i^{\Lambda_i}(\theta_i) > \max_{j \in \mathcal{E}(\theta) \setminus \{i\}} \{(v_j - \phi_j^{\Lambda_j}(\theta_j))^+\}, \text{ and } i \in \mathcal{E}(\theta) \\ 0, & \text{if } v_i - \phi_i^{\Lambda_i}(\theta_i) < \max_{j \in \mathcal{E}(\theta) \setminus \{i\}} \{(v_j - \phi_j^{\Lambda_j}(\theta_j))^+\}, \text{ or } i \notin \mathcal{E}(\theta) \end{cases},$$

and

$$r_i(\theta) = \mathbf{1}\{i \in \mathcal{E}(\theta)\}.$$

where $\phi_i^{\Lambda_i}$ is implied by Lemma 2. In particular, since $\phi_i^{\Lambda_i}$ is nondecreasing for all i, $\theta_i \mapsto r_i(\theta_i, \theta_{-i})$ is nondecreasing for all i. We now claim that $\boldsymbol{p}_i(\theta) \leq \phi_i^{\Lambda_i}(\theta_i)$ for all i and for G-almost all θ such that $r_i(\theta) > 0$. Indeed, suppose the contrary, that $\boldsymbol{p}_i(\theta) > \phi_i^{\Lambda_i}(\theta_i)$ for some i and for a positive G measure of θ. Then, since F has full support and since $\underline{\theta}_i = \phi_i^{\Lambda_i}(\underline{\theta}_i) \leq v_i$ with positive F-measure, there is a positive F-measure of $\mathbf{v}$ and a positive G-measure of θ such that $\phi_i^{\Lambda_i}(\theta_i) < v_i < \boldsymbol{p}_i(\theta)$ and that $v_j \leq \phi_j(\theta_j)^{\Lambda_j}$ for all $j \neq i$. For these $\mathbf{v}$ and θ, $\mu_i(\mathbf{v}|\theta) = 1 > 0$ but $v_i < \boldsymbol{p}_i(\theta)$, a contradiction.

Now let
$$q_i(\theta) := r_i(\theta) \int_V \boldsymbol{\mu}_i(\mathbf{v}|\theta) F(\mathrm{d}\mathbf{v}),$$
for all i and for all $\theta \in \Theta$. Lemma 1 then implies that
$$\mathbb{E}_{\theta_{-i}}[\boldsymbol{p}_i(\theta) q_i(\theta)] = \theta_i(Q_i(\theta_i) + R_i(\theta_i)\kappa_i) + \int_{\theta_i}^{\bar{\theta}_i} (Q_i(x) + R_i(x)\kappa_i)\,\mathrm{d}x,$$
where $Q_i(\theta_i) := \mathbb{E}_{\theta_{-i}}[q_i(\theta)]$ and $R_i(\theta_i) := \mathbb{E}_{\theta_{-i}}[r_i(\theta)]$. As $\boldsymbol{p}_i(\theta) \leq \phi_i^{\Lambda_i}(\theta_i)$ for all i and for G-almost all θ such that $r_i(\theta) > 0$,
$$\mathbb{E}_{\theta_{-i}}[\boldsymbol{p}_i(\theta) q_i(\theta)] \leq \phi_i^{\Lambda_i}(\theta_i) Q_i(\theta_i).$$
Thus, for any i and for any θ such that $r_i(\theta) > 0$,
$$\theta_i(Q_i(\theta_i) + R_i(\theta_i)\kappa_i) + \int_{\theta_i}^{\bar{\theta}_i} (Q_i(x) + R_i(x)\kappa_i)\,\mathrm{d}x \leq \phi_i^{\Lambda_i}(\theta_i) Q_i(\theta_i).$$
Moreover, since $\theta_i \mapsto r_i(\theta_i, \theta_{-i})$ is nondecreasing, for any i and for any θ such that $r_i(\theta) > 0$, it must be that $r_i(\underline{\theta}_i, \theta_{-i}) > 0$. Together with the fact that $\phi_i^{\Lambda_i}(\underline{\theta}_i) = \underline{\theta}_i$ (see Theorem 4 of Monteiro and Svaiter, 2010), it must be that
$$\underline{\theta}_i R_i(\underline{\theta}_i)\kappa_i + \int_{\underline{\theta}_i}^{\bar{\theta}_i} (Q_i(x) + R_i(x)\kappa_i)\,\mathrm{d}x \leq 0,$$
which in turn, by condition 2 of Lemma 1, implies that $Q_i(\theta_i) = 0$ for all $\theta_i > \underline{\theta}_i$. Hence, $q_i(\theta_i, \theta_{-i}) = 0$ for all i, for all $\theta_i > \underline{\theta}_i$ and for G_{-i}-almost all θ_{-i}. Therefore, since F has full support on V and since, $v_i \geq \underline{\theta}_i = \phi_i^{\Lambda_i}(\underline{\theta}_i)$ with positive F-measure for every i, it must be that $\mathrm{supp}(G_i) = \{\underline{\theta}_i\}$ for all i. This completes the proof.

■